CMSC 435
Introductory Computer Graphics

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UMBC

Announcements

- Proj1
  - Due tomorrow

Translation

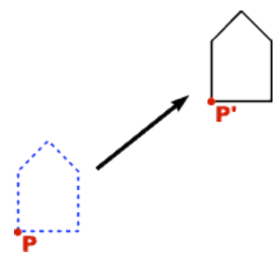
- Offset from position

  $x' = x + t_x$

  $y' = y + t_y$

- In matrix form

  $P' = P + T$



  where $T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$

- Repeat transformation for each point in figure (rigid body transformation)

Scaling

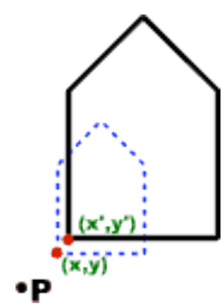
- Resize relative to position

  $x' = x * s_x$

  $y' = y * s_y$

- In matrix form

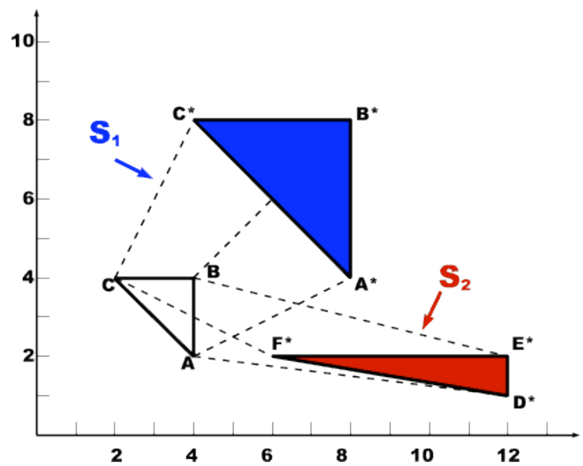
  $P' = P * S$



  where $T = \begin{bmatrix} s_x \\ s_y \end{bmatrix}$

- Repeat transformation for each point in figure (rigid body transformation)

Non-uniform Scaling



Rotation

- Turn about a point

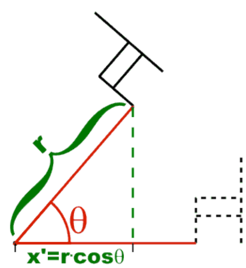
  x' = x cosθ - y sinθ

  y' = x sinθ + y cosθ

- In matrix form

  P' = R•P

  where R = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$



Homogenous Coordinates

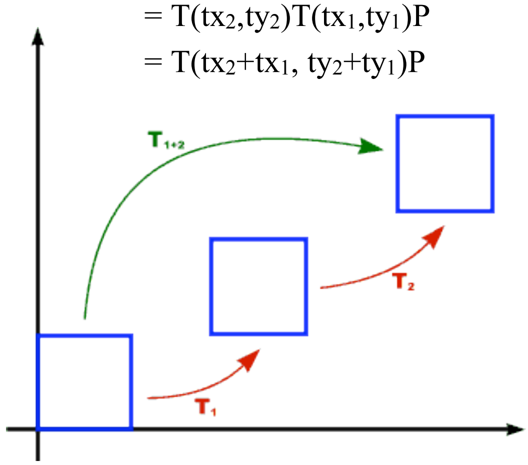
- Want to do all standard transformations as matrix multiplications

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x/h \\ y/h \\ 1 \end{bmatrix} or \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$

Composite Transformations

- Translation

$$\begin{aligned} P' &= T_2 T_1 P \\ &= T(tx_2,ty_2)T(tx_1,ty_1)P \\ &= T(tx_2+tx_1,\ ty_2+ty_1)P \end{aligned}$$

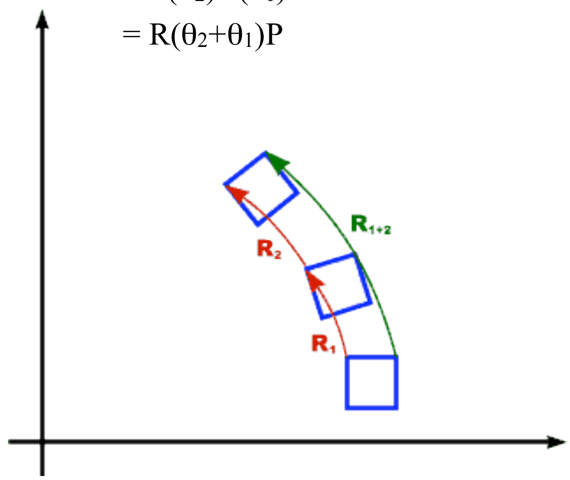


Composite Transformations

- Rotation

  $P' = R(\theta_2)R(\theta_1)P$

  $= R(\theta_2+\theta_1)P$



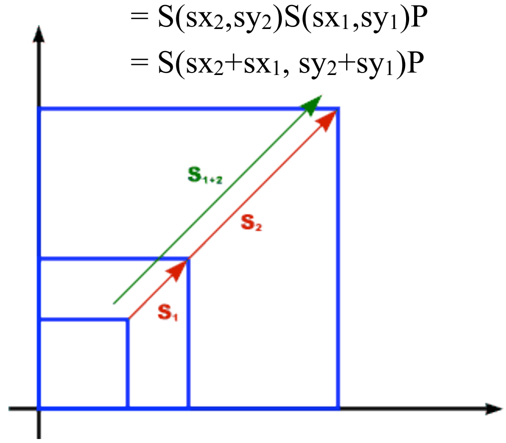
Composite Transformations

- Scaling

  $P' = S_2S_1P$

  $= S(sx_2,sy_2)S(sx_1,sy_1)P$

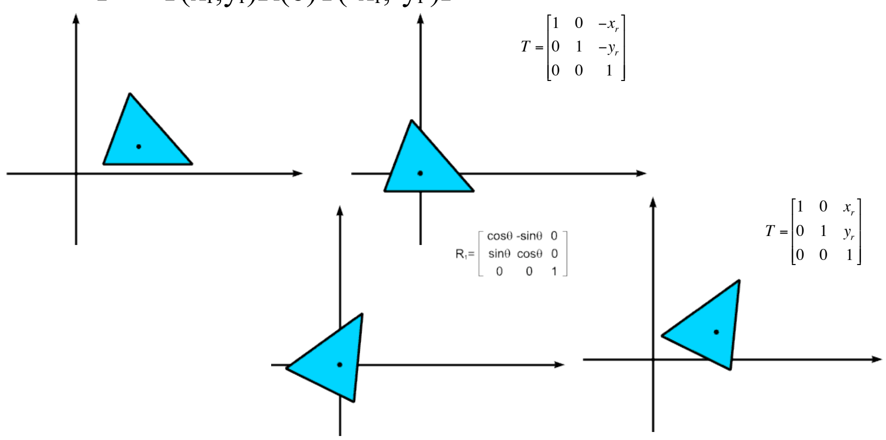
  $= S(sx_2+sx_1, sy_2+sy_1)P$



General Pivot Point Rotation

- A transformation sequence for rotating an object about a specified pivot point

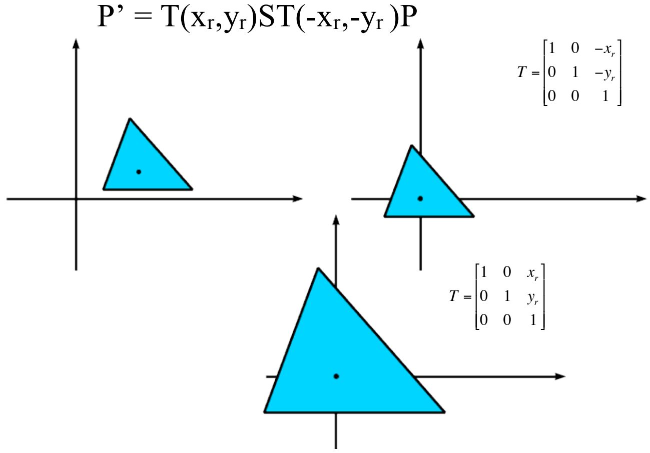
  $P' = T(x_r, y_r)R(\theta)T(-x_r, -y_r)P$



General Pivot Point Scaling

- A transformation sequence for scaling an object about a specified pivot point

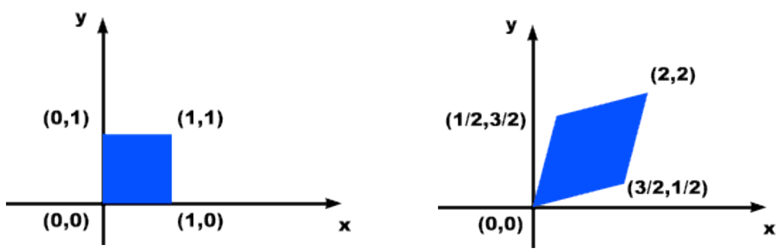
  $P' = T(x_r, y_r)ST(-x_r, -y_r)P$



General Scaling Direction

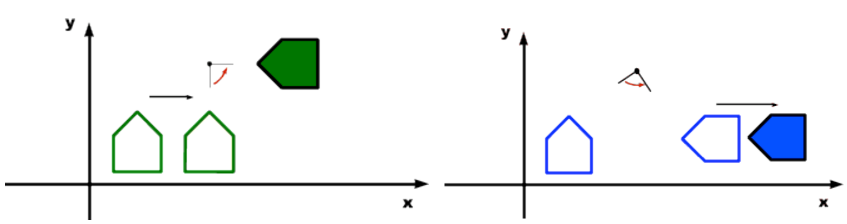
- A transformation sequence for scaling an object along a specified direction
  - Rotate scaling direction to axis
  - Scale
  - Rotate back

  P’ = R(θ)SR(-θ)P

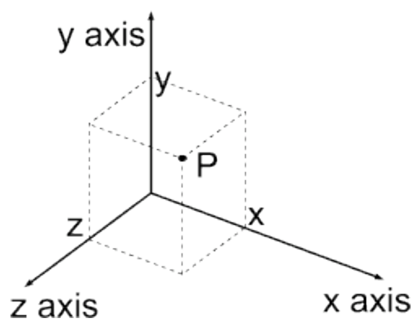


Order of Transformations

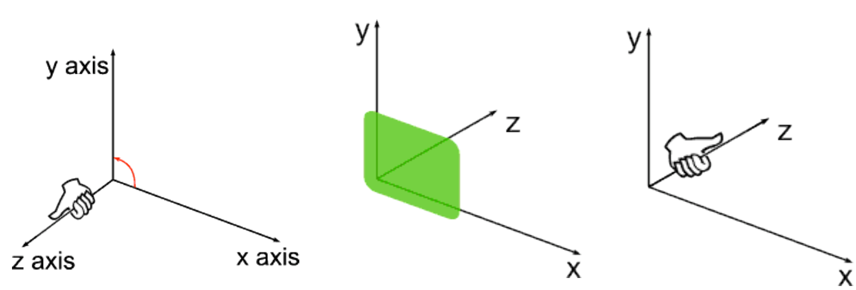
- RTP != TRP



3D Cartesian Coordinate Systems



Handedness

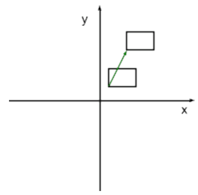


Translation

- 2D Translation

  $x' = x + t_x$

  $y' = y + t_y$



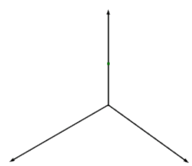
$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- 3D Translation

  $x' = x + t_x$

  $y' = y + t_y$

  $z' = z + t_z$



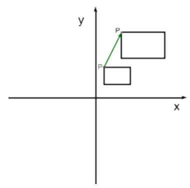
$$T = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling

- 2D Translation

  $x' = x * s_x$

  $y' = y * s_y$



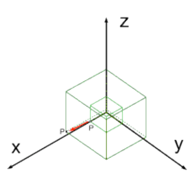
$$S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- 3D Translation

  $x' = x * s_x$

  $y' = y * s_y$

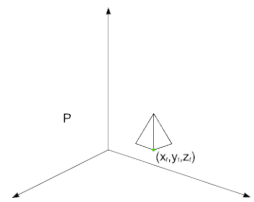
  $z' = z * s_z$

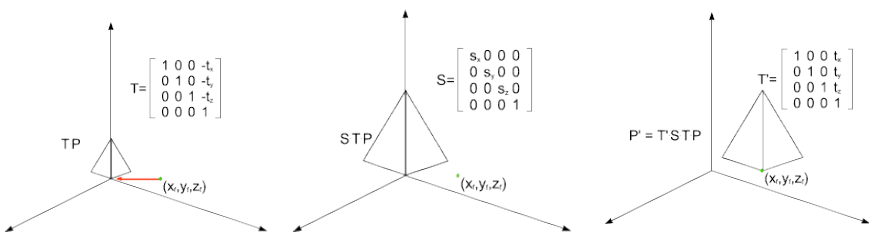


$$S = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

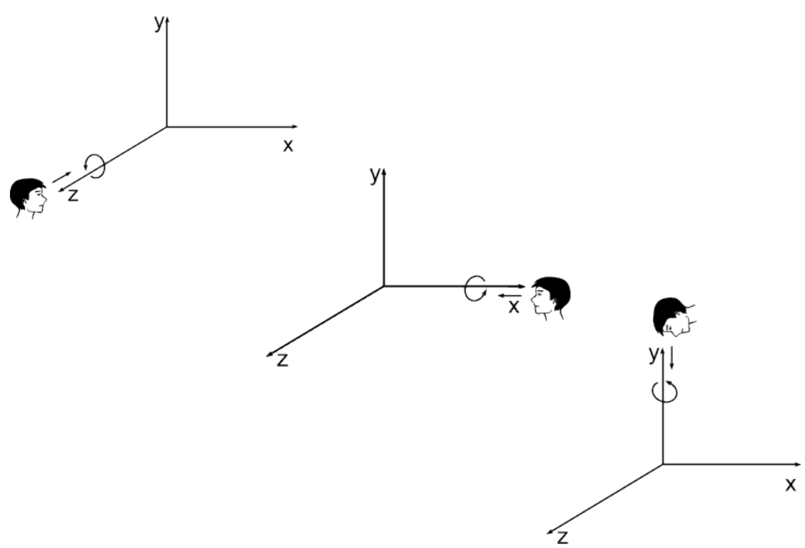
Scale with Fixed Point

- Translate fixed point to origin
- Scale
- Translate back

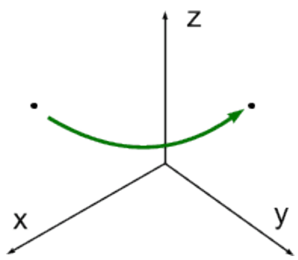




Positive 3D Rotation



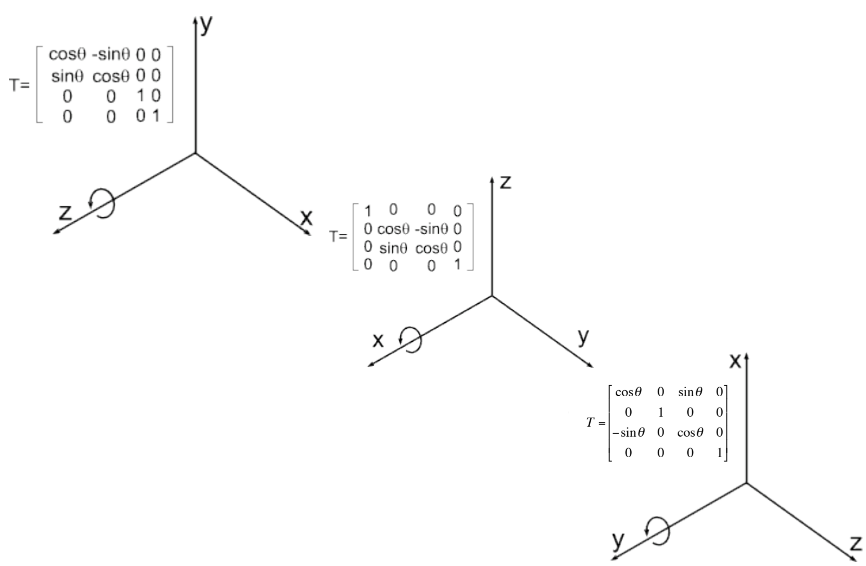
Rotation about Z axis



- Like 2D
  x’ = x cosθ - y sinθ
  y’ = x sinθ + y cosθ
  z’ = z

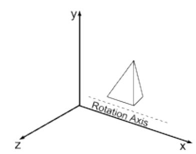
$$T=\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

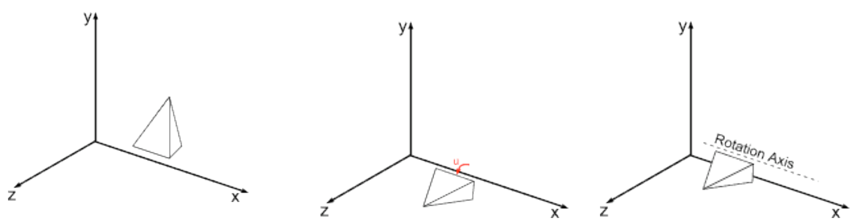
Rotation about Coordinate Axes



Rotation about Axis Parallel to X

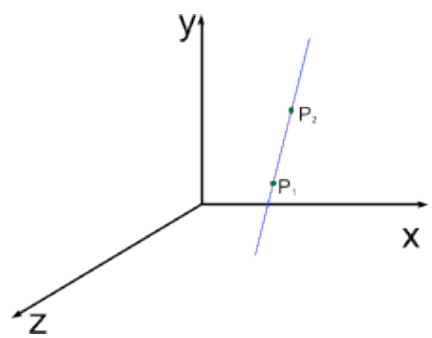
- Translate rotation axis to X
- Rotate
- Translate back





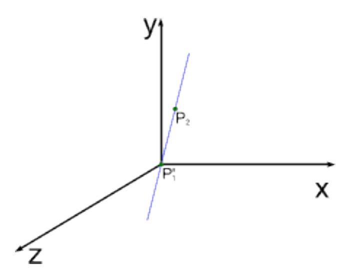
Rotation about Arbitrary Axis

- To rotate about axis through P_1P_2

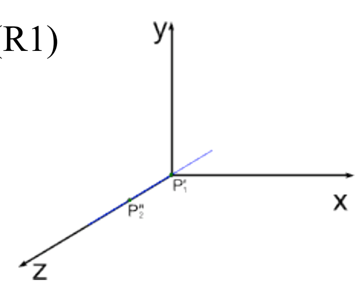


Rotation about Arbitrary Axis (2)

- Translate P_1 to origin (T)

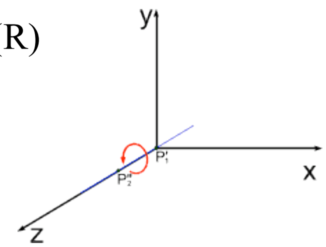


- Rotate until P_2 lies on z axis (R1)

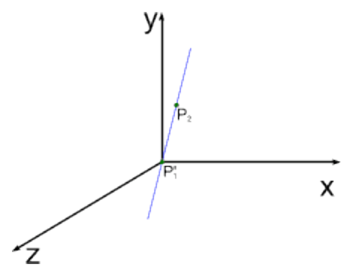


Rotation about Arbitrary Axis (3)

- Perform desired rotation (R)

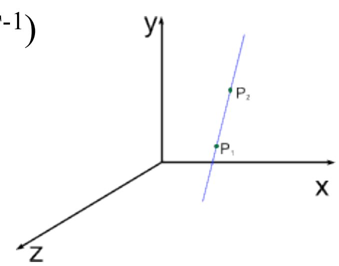


- Rotate axis back (R_1^{-1})



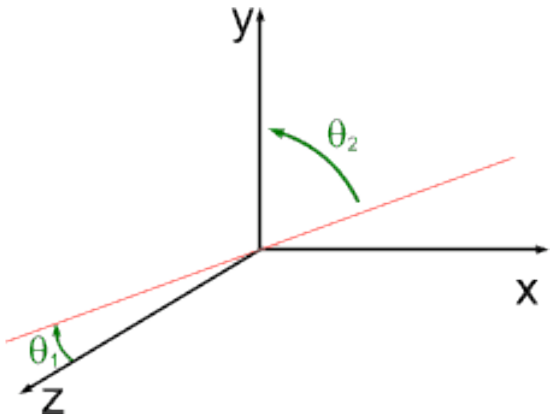
Rotation about Arbitrary Axis (4)

- Translate axis back (T^{-1})



Rotation Axis Specification

1. Point and two rotation angles



Rotation Axis Specification

2. Two points

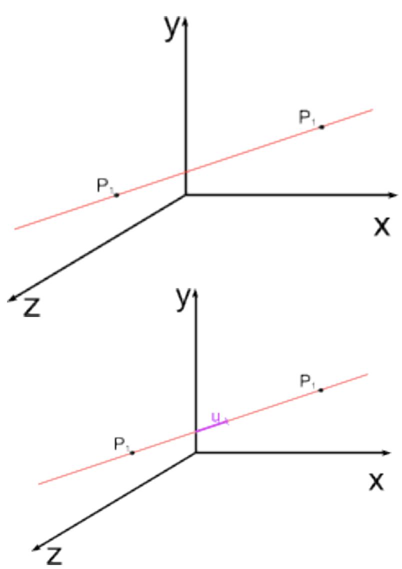
$V = P_2 - P_1$

$U = V / |V| = (a,b,c)$

$a = (x_2 - x_1) / |V|$

$b = (y_2 - y_1) / |V|$

$c = (z_2 - z_1) / |V|$



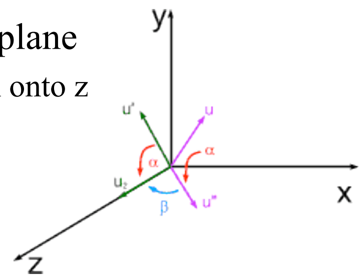
Rotating Axis onto Coordinate Axis

1. Rotate about x-axis into xz plane

   Same as rotation of yz projection onto z

   u' = (0,b,c)

   u_2 = (0,0,d)



Rotating Axis onto Coordinate Axis

1. Rotate about x-axis into xz plane

   Same as rotation of yz projection onto z

   u' = (0,b,c)

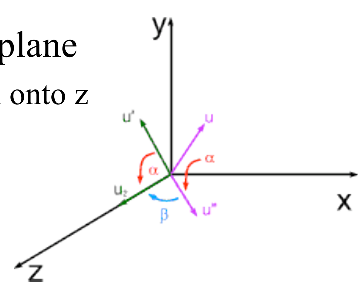
   u_2 = (0,0,d)

   $\cos \alpha = (u' \bullet u_2)/(|u'||u_2|$

   $= cd/dd$

   $= c/d$

   where $d = \text{sqrt}(b^2+c^2)$



Rotating Axis onto Coordinate Axis

1. Rotate about x-axis into xz plane

Same as rotation of yz projection onto z

$u' = (0,b,c)$

$u_2 = (0,0,d)$

$\cos\alpha = (u' \bullet u_2)/(|u'||u_2|$

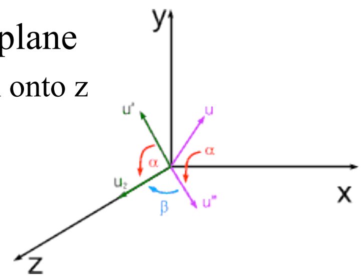
$= cd/dd$

$= c/d$

where $d = \text{sqrt}(b^2+c^2)$

$\sin\alpha = b/d$

$u'' = (a,0,d)$



$$R_x(x)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotating Axis onto Coordinate Axis

1. Rotate about x-axis into xz plane
2. Rotate about y onto x axis

$$R_x(x)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(x)=\begin{bmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- composite rotation $R_1 = R_y(\beta)R_x(\alpha)$