CMSC 435
Introductory Computer Graphics
Pipeline

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Announcements

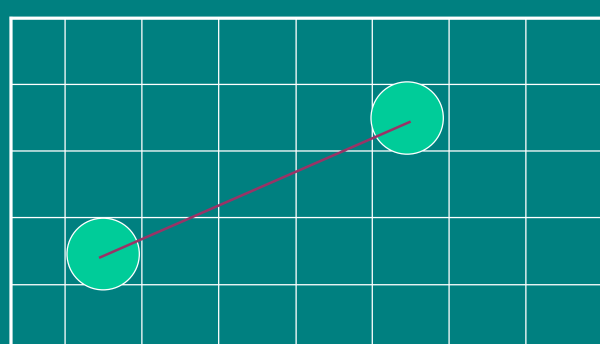
- Wed-Sat on travel
  - Limited email access
  - Guest lecture Thurs by Wes Griffin on OpenGL
- Project 2
  - Status/issues

Graphics Pipeline

- Object-order approach to rendering
- Sequence of operations
  - Vertex processing
    - Transforms
    - Viewing
    - Vertex components of shading/texture
  - Rasterization
    - Break primitives into fragments/pixels
    - Clipping
  - Fragment processing
    - Fragment components of shading/texture
  - Blending

Line Drawing

- Given endpoints of line, which pixels to draw?



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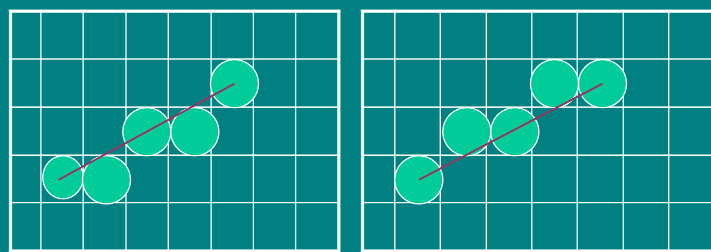

Line Drawing

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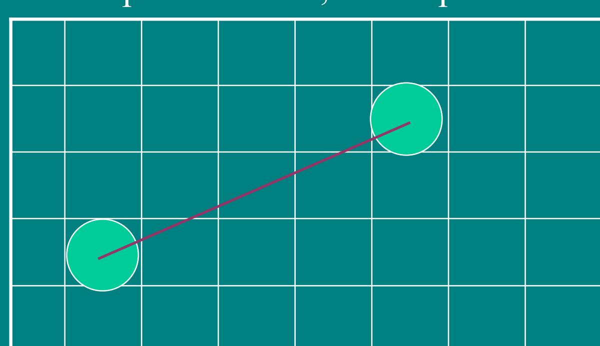


- Assume one pixel per column (x index), which row (y index)?
- Choose based on relation of line to midpoint between candidate pixels

Line Drawing

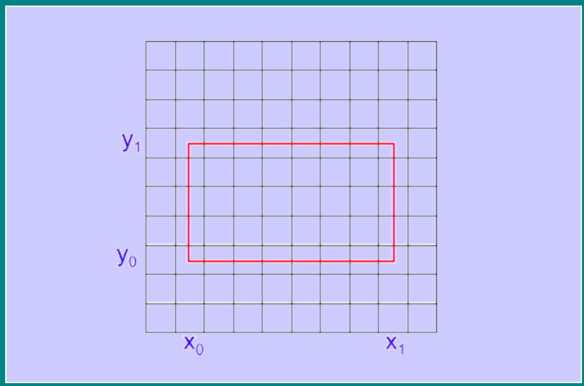
- Implicit representation
  - $f(x,y)=(y_0-y_1)x + (x_1-x_0)y + x_0y_1 - x_1y_0 = 0$
  - Slope $m = (y_1-y_0)/(x_1-x_0)$ (assume $0 \le m \le 1$)
- Midpoint algorithm

```
y=y0
d = f(x0+1, y0+0.5)
for x = x0 to x1 do
   draw (x,y)
   if (d < 0) then
      y = y+1
      d = d + (x1 - x0) + (y0 - y1)
   else
      d = d + (y0 - y1)
```

Scan conversion

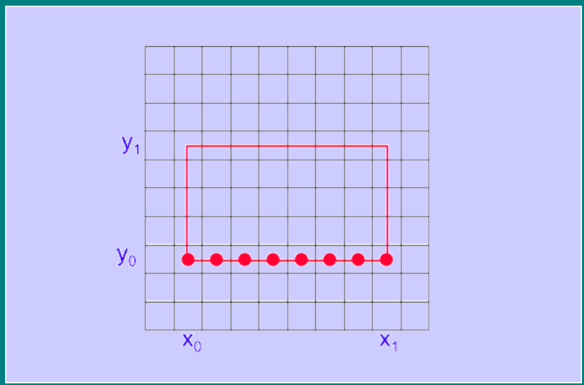
- Problem
  - How to generate filled polygons (by determining which pixel positions are inside the polygon)
  - Conversion from continuous to discrete domain
- Concepts
  - Spatial coherence
  - Span coherence
  - Edge coherence

Scanning Rectangles



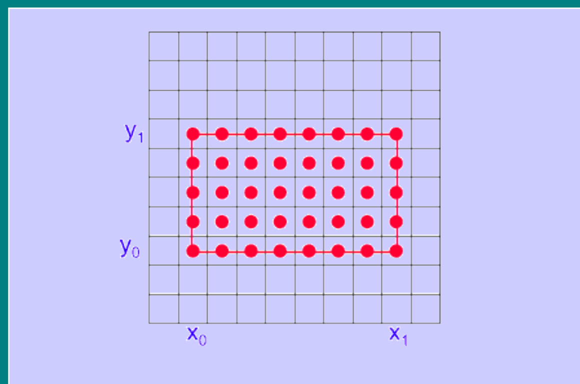

```
for ( y from y0 to yn )
   for ( x from x0 to xn )
      Write Pixel (x, y, val)
```

Scanning Rectangles (2)




```
for ( y from y0 to yn )
   for ( x from x0 to xn )
      Write Pixel (x, y, val)
```

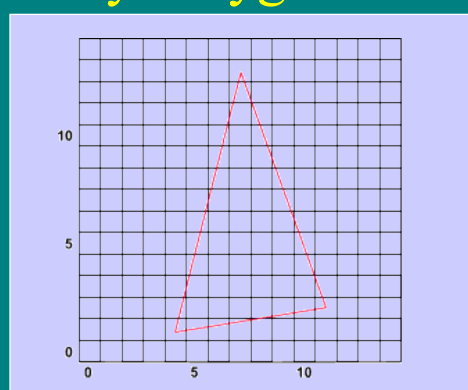
Scanning Rectangles (3)




```
for ( y from y0 to yn )
   for ( x from x0 to xn )
      Write Pixel (x, y, val)
```

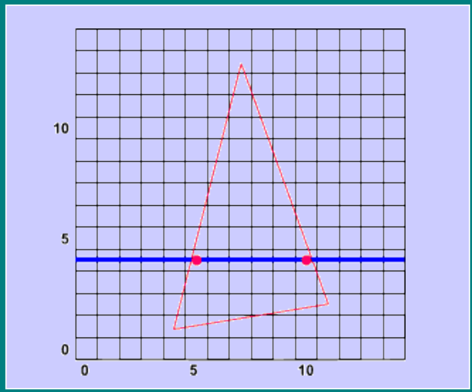
Scanning Arbitrary Polygons

- vertices:
  (4, 1) , (7, 13) , (11 , 2)



Scanning Arbitrary Polygons (2)

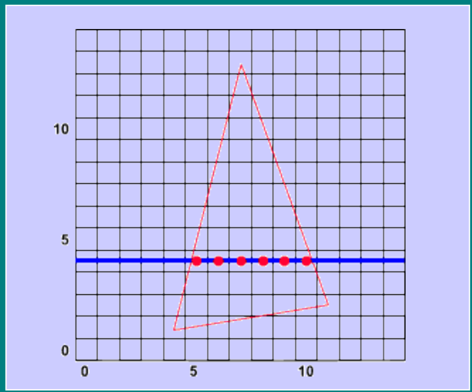
- vertices:
  (4, 1) , (7, 13) , (11 , 2)



- Intersect scanline w/pgon edges => span extrema

Scanning Arbitrary Polygons (3)

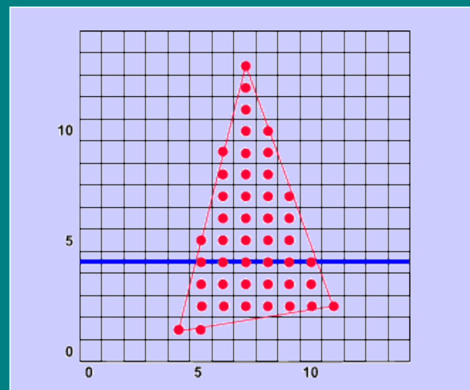
- vertices:
  (4, 1) , (7, 13) , (11 , 2)



- Intersect scanline w/pgon edges => span extrema
- Fill between pairs of span extrema

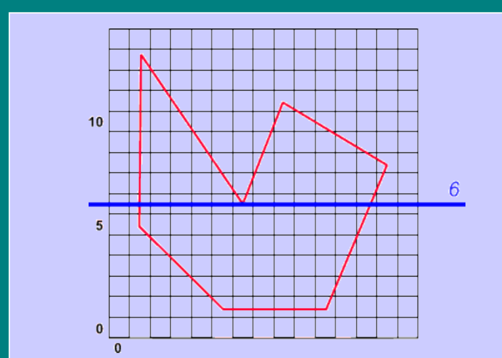
Scanning Arbitrary Polygons (4)

- vertices:
  (4, 1) , (7, 13) , (11 , 2)



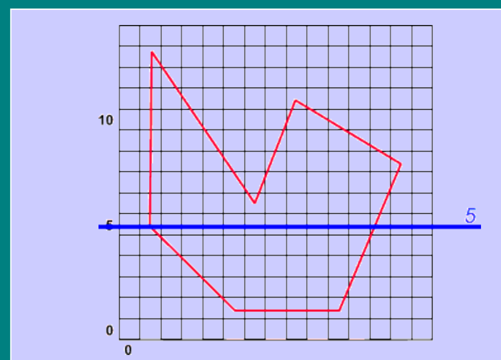
For each nonempty scanline
Intersect scanline w/pgon edges => span extrema
Fill between pairs of span extrema

Example Cases (2)



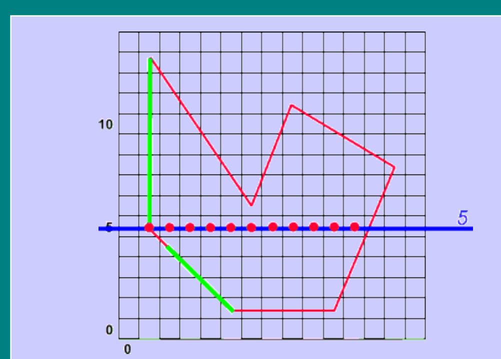
4 intersections w/ scanline 6 at x = 1, 6, 6, 12 1/7

Example Cases (3)



- 3 intersections w/scanline 5 at x = 1, 1, 11 5/7

Example Cases (4)

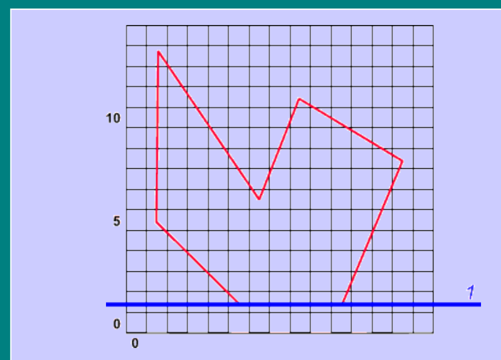


3 intersections w/scanline 5 at x = 1, 1, 11 5/7

==>

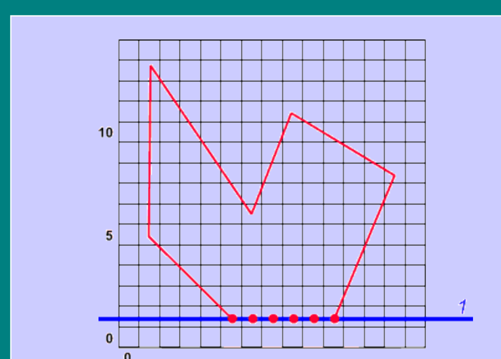
Count continuing edges once (shorten lower edge) now x=1, 11 5/7

Example Cases (5)



4 intersections w/ scanline 1at x = 5, 5, 10, 10

Example Cases (6)



4 intersections w/ scanline 1 at x = 5, 5, 10, 10

=>

Don't count vertices of horizontal edges.

Now x = 5, 10

Scanline Data Structures

Sorted edge table:

all edges

sorted by min y

holds:

max y

init x

inverse slope

Active edge table:

edges intersecting current scanline

holds:

max y

current x

inverse slope

Scanline Algorithm

1. Bucket sort edges into sorted edge table
2. Initialize y & active edge table

    y = first non- empty scanline

    AET = SET [y]
3. Repeat until AET and SET are empty

    Fill pixels between pairs of x intercepts in AET

    Remove exhausted edges

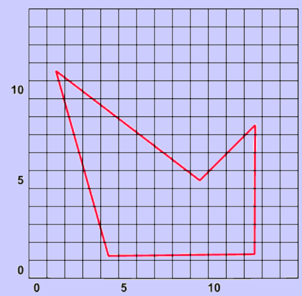
    Y++

    Update x intercepts

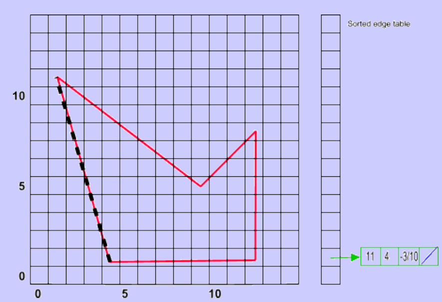
    Resort table (AET)

    Add entering edges

Example: vertices (4,1), (1,11), (9,5), (12,8), (12,1)



Example: vertices (4,1), (1,11), (9,5), (12,8), (12,1)



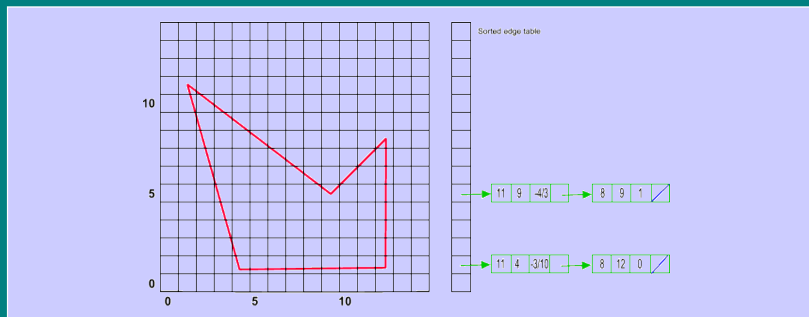
bucket sort edges into sorted edge table

sort on minY: 1

store:

- max Y: 11
- min X: 4
- 1/m : (Xmax - Xmin) / (Ymax - Ymin) = (1 - 4) / (11 - 1) = -3 / 10

Example: vertices (4,1), (1,11), (9,5), (12,8), (12,1)



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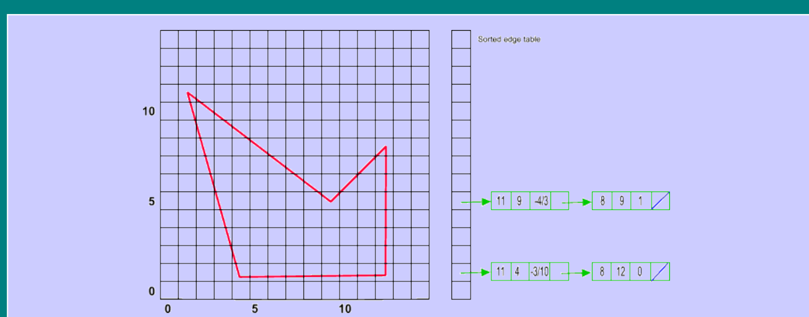
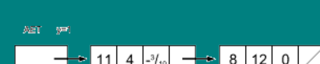


bucket sort edges into sorted edge table
initialize active edge list to first non empty scanline



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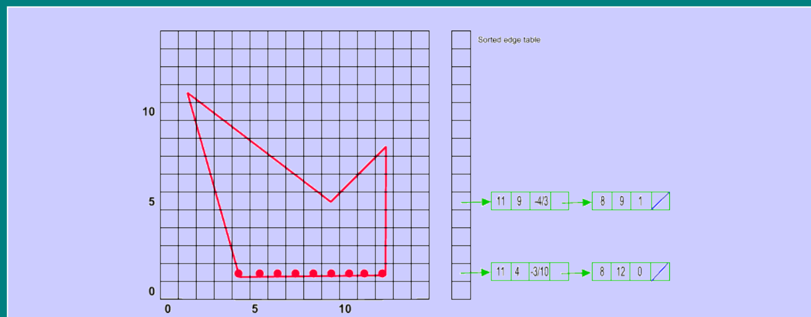
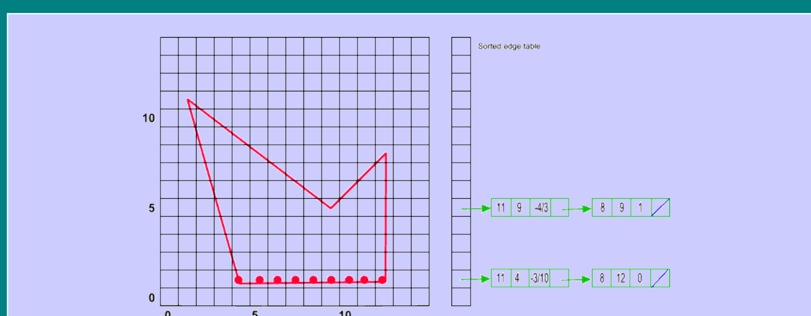
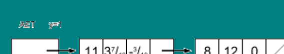


bucket sort edges into sorted edge table
initialize active edge list to first non empty scanline
for each non empty scanline
  fill between pairs (x=4,12)

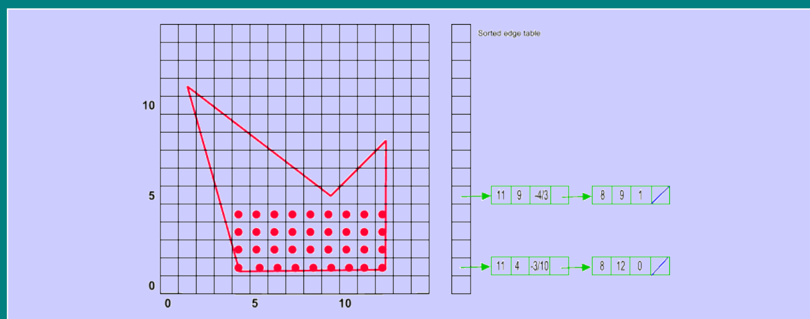
Example: vertices (4,1), (1,11), (9,5), (12,8), (12,1)



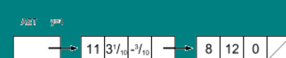
bucket sort edges into sorted edge table
initialize active edge list to first non empty scanline
for each non empty scanline
  fill between pairs (x=4,12)
  remove exhausted edges
  update intersection points
  resort table
  add entering edges



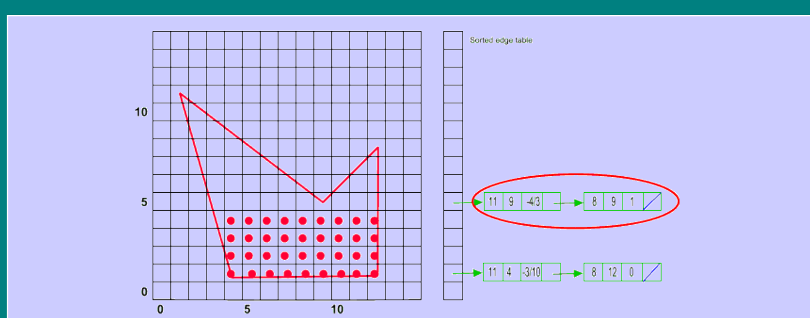
Example: vertices (4,1), (1,11), (9,5), (12,8), (12,1)



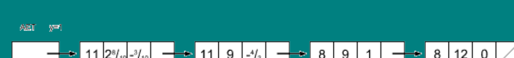
bucket sort edges into sorted edge table
initialize active edge list to first non empty scanline
for each non empty scanline
  fill between pairs (x=3 1/10,12)
  remove exhausted edges
  update intersection points



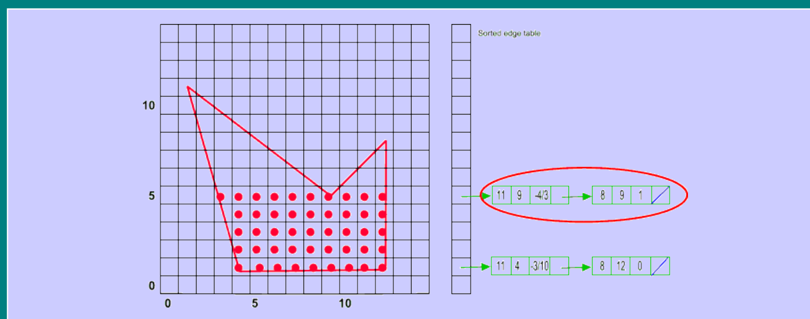
Example: vertices (4,1), (1,11), (9,5), (12,8), (12,1)



bucket sort edges into sorted edge table
initialize active edge list to first non empty scanline
for each non empty scanline
  fill between pairs (x=3 1/10,12)
  remove exhausted edges
  update intersection points
  resort table
  add entering edges

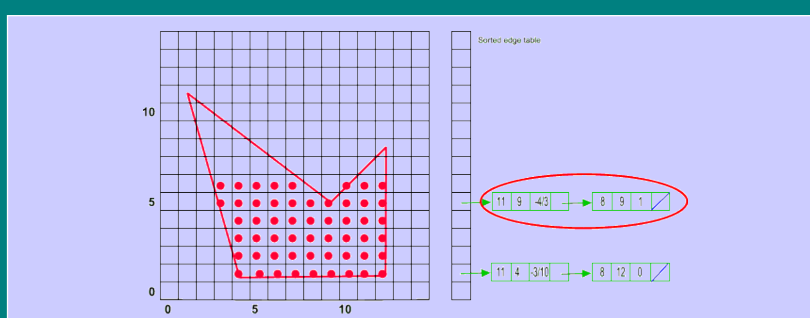


Example: vertices (4,1), (1,11), (9,5), (12,8), (12,1)

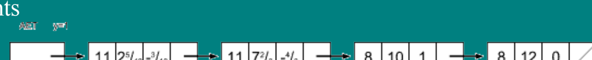


bucket sort edges into sorted edge table
initialize active edge list to first non empty scanline
for each non empty scanline
  fill between pairs (x = 2 8/10, 9; 9,12)
  remove exhausted edges
  update intersection points
  resort table
  add entering edges

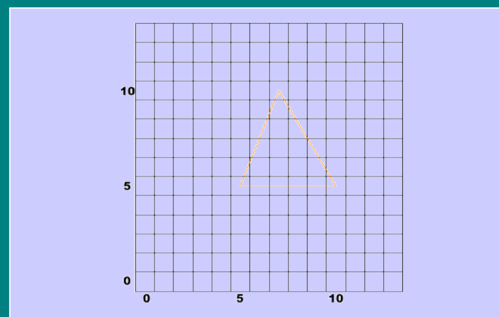
Example: vertices (4,1), (1,11), (9,5), (12,8), (12,1)



bucket sort edges into sorted edge table
initialize active edge list to first non empty scanline
for each non empty scanline
  fill between pairs (x=2 5/10, 7 2/3; 10,12)
  remove exhausted edges
  update intersection points
  resort table
  add entering edges



Fill Variants

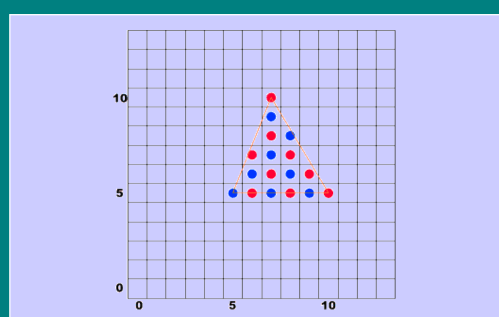


Fill between pairs:

```
for ( x = x1; x < x2; x++ )
     framebuffer [ x, y ] = c
```

Fill Variants (2)

- Pattern Fill

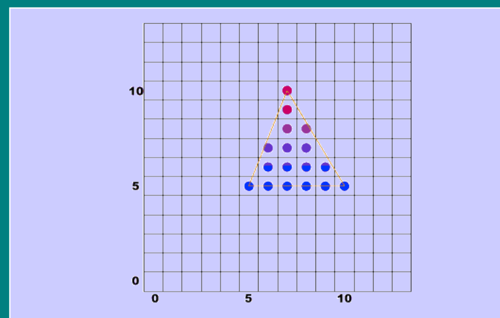


Fill between pairs:

```
for ( x = x1; x < x2; x++ )
      if ( ( x + y ) % 2 )
         framebuffer [ x, y ] = c1
      else
         framebuffer [ x, y ] = c2
```

Fill Variants (3)

- Colorwash
  - Red to blue



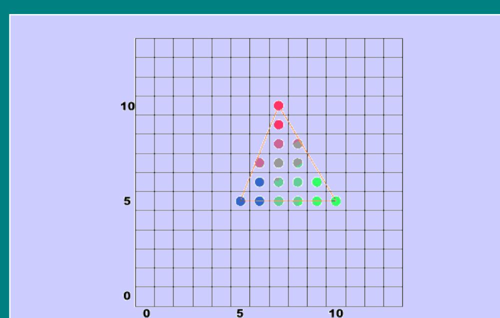
Fill between pairs:

```
for ( x = x1; x < x2; x++ )
    framebuffer [ x, y ] = C0 + dC * ( x1 - x )
```

For efficiency carry C and dC in AETand calculate color incrementally

Fill Variants (4)

- Vertex colors
  - Red, green, blue



Fill between pairs:

```
for ( x = x1; x < x2; x++ )
  framebuffer [ x, y ] =
     Cy1x1 + [(x - x1)/(x2 - x1)*(Cy1x2 - Cy1x1)]/dCx
```

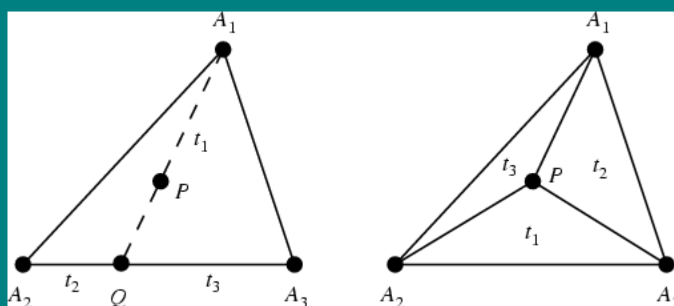
For efficiency carry Cy and dCy in AET calculate dCx at beginning of scanline

Barycentric Coordinates

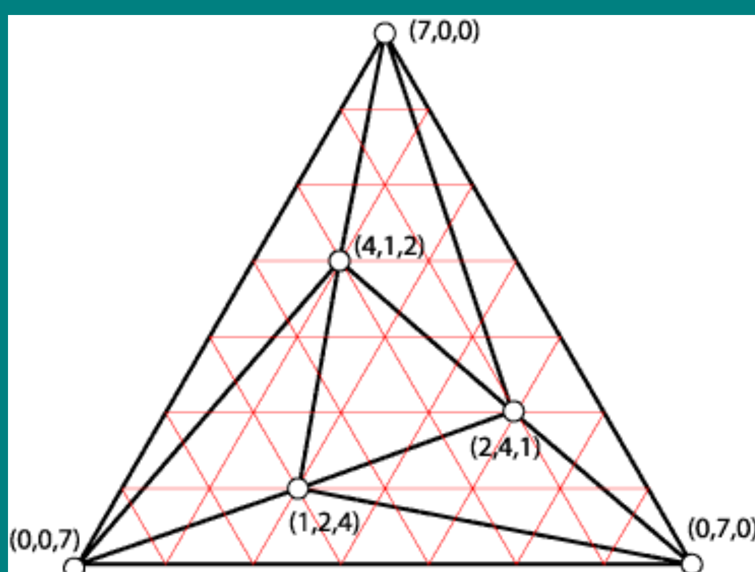
- Use non-orthogonal coordinates to describe position relative to vertices

$$p = a + \beta(b - a) + \gamma(c - a) \qquad p(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$$

  - Coordinates correspond to scaled signed distance from lines through pairs of vertices



Barycentric Example



Barycentric Coordinates

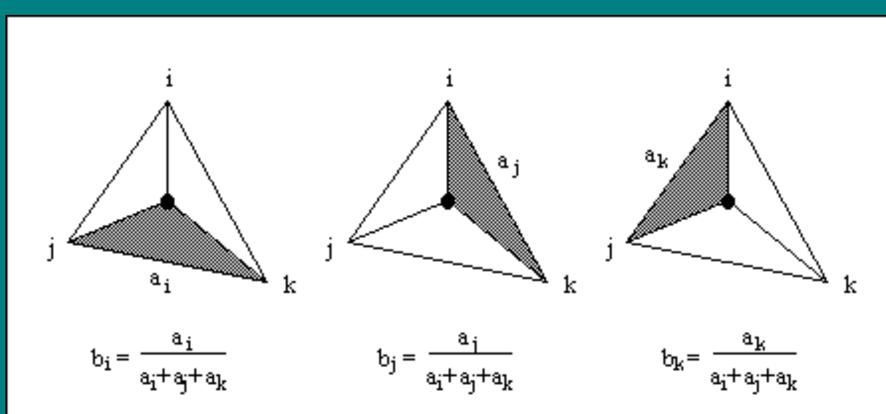
- Computing coordinates

$$\gamma = \frac{(y_a - y_b)x + (x_b - x_a)y + x_a y_b - x_b y_a}{(y_a - y_b)x_c + (x_b - x_a)y_c + x_a y_b - x_b y_a}$$

$$\beta = \frac{(y_a - y_c)x + (x_c - x_a)y + x_a y_c - x_c y_a}{(y_a - y_c)x_b + (x_c - x_a)y_b + x_a y_c - x_c y_a}$$

$$\alpha = 1 - \beta - \gamma$$

Alternative Computation



Barycentric Rasterization

For all x do
    For all y do
        Compute (α, β, γ) for (x,y)
        If ($\alpha \in [0,1]$ and $\beta \in [0,1]$ and $\gamma \in [0,1]$) then
            $c = \alpha c_0 + \beta c_1 + \gamma c_2$
            Draw pixel (x,y) with color c

Barycentric Rasterization

x_{min} = floor(x_i)
x_{max} = ceiling(x_i)
y_{min} = floor(y_i)
y_{max} = ceiling(x_i)
for y = y_{min} to y_{max} do
    for x = x_{min} to x_{max} do
        $\alpha = f_{12}(x,y)/f_{12}(x_0,y_0)$
        $\beta = f_{20}(x,y)/f_{20}(x_1,y_1)$
        $\gamma = f_{01}(x,y)/f_{01}(x_2,y_2)$
        If ($\alpha \in [0,1]$ and $\beta \in [0,1]$ and $\gamma \in [0,1]$) then
            $c = \alpha c_0 + \beta c_1 + \gamma c_2$
            Draw pixel (x,y) with color c

Barycentric Rasterization

- Computing coordinates

$$\gamma = \frac{f_{01}(x,y)}{f_{01}(x_2,y_2)} = \frac{(y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0}{(y_0 - y_1)x_2 + (x_1 - x_0)y_2 + x_0y_1 - x_1y_0}$$

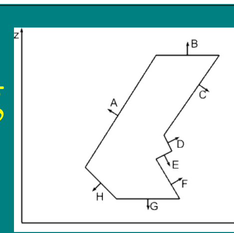
$$\beta = \frac{f_{20}(x,y)}{f_{20}(x_1,y_1)} = \frac{(y_2 - y_0)x + (x_0 - x_2)y + x_2y_0 - x_0y_2}{(y_2 - y_0)x_1 + (x_0 - x_2)y_1 + x_2y_0 - x_0y_2}$$

$$\alpha = \frac{f_{12}(x,y)}{f_{12}(x_0,y_0)} = \frac{(y_1 - y_2)x + (x_2 - x_1)y + x_1y_2 - x_2y_1}{(y_1 - y_2)x_0 + (x_2 - x_1)y_0 + x_1y_2 - x_2y_1}$$

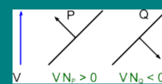
Visibility

- We can convert simple primitives to pixels/fragments
- How do we know which primitives (or which parts of primitives) should be visible?

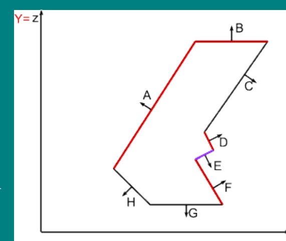
Back-face Culling



- Polygon is back-facing if
  - V•N > 0

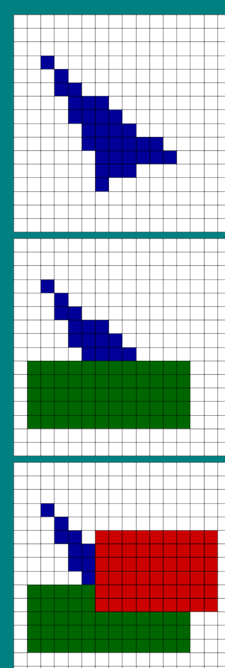


- Assuming view is along Z (V=0,0,1)
  - V•N + (0 + 0 + z_n)
- Simplifying further
  - If $z_n > 0$, then cull
- Works for non-overlapping convex polyhedra
- With concave polyhedra, some hidden surfaces will not be culled



Painter's Algorithm

- First polygon:
  - (6,3,10), (11, 5,10), (2,2,10)
- Second polygon:
  - (1,2,8), (12,2,8), (12,6,8), (1,6,8)
- Third polygon:
  - (6,5,5), (14,5,5), (14,10,5),(6,10,5)



Painter's Algorithm

- Given

  List of polygons {P_1, P_2, …. P_n)

  An array of Intensity [x,y]

- Begin

  Sort polygon list on minimum Z (largest z-value comes first in sorted list)

  For each polygon P in selected list do

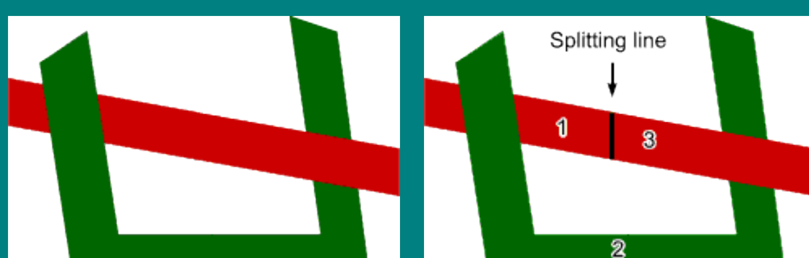
      For each pixel (x,y) that intersects P do

          Intensity[x,y] = intensity of P at (x,y)

  Display Intensity array

Painter's Algorithm: Cycles

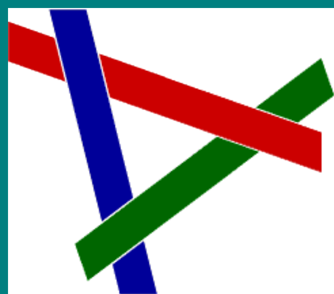
- Which order to scan?

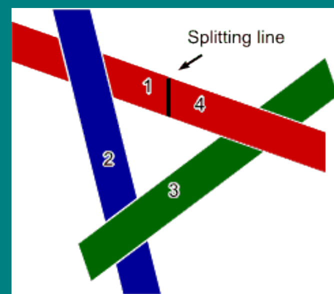


- Split along line, then scan 1,2,3

Painter's Algorithm: Cycles

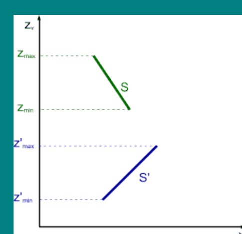
- Which to scan first?





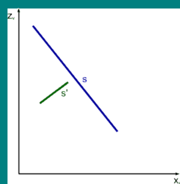
- Split along line, then scan 1,2,3,4 (or split another polygon and scan accordingly)
- Moral: Painter's algorithm is fast and easy, except for detecting and splitting cycles and other ambiguities

Depth-sort: Overlapping Surfaces

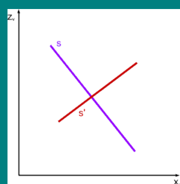


- Assume you have sorted by maximum Z
  - Then if $Z_{min} > Z'_{max}$, the surfaces do not overlap each other (minimax test)
- Correct order of overlapping surfaces may be ambiguous. Check it.

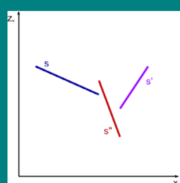
Depth-sort: Overlapping Surfaces



- No problem: paint S, then S’

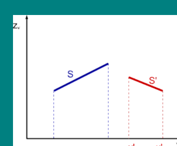


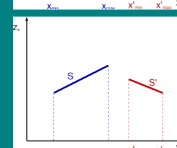
- Problem: painting in either order gives incorrect result



- Problem? Naïve order S S’ S”; correct order S’ S” S

Depth-sort: Order Ambiguity

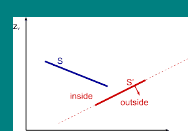




1. Bounding rectangles in xy plane do not overlap
   - Check overlap in x

     $x'_{min} > x_{max}$ or $x_{min} > x'_{max}$ -> no overlap
   - Check overlap in y

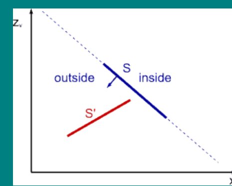
     $y'_{min} > y_{max}$ or $y_{min} > y'_{max}$ -> no overlap



2. Surface S is completely behind S’ relative to viewing direction.
   - Substitute all vertices of S into plane equation for S’, if all are “inside” (< 0), then there is no ambiguity

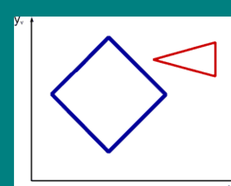
Depth-sort: Order Ambiguity

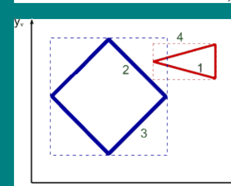
3. Surface S’ is completely in front S relative to viewing direction.
   - Substitute all vertices of S’ into plane equation for S, if all are “outside” (>0), then there is no ambiguity

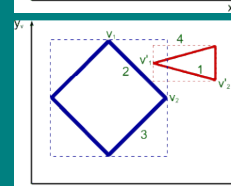


Depth-sort: Order Ambiguity

4. Projection of the two surfaces onto the viewing plane do not overlap
   - Test edges for intersection
   - Rule out some pairs with minimax tests (can eliminate 3-4 intersection, but not 1-2)
   - Check slopes -- parallel lines do not intersect
   - Compute intersection points:
     - $s = [(x'_1-x'_2)(y_1-y'_1) - (x_1-x'_1)(y'_1-y'_2)]/D$
     - $t = [(x_1 - x_2)(y_1 - y'_1) - (x_1 - x'_1)(y_1 - y_2)]/D$
     - $D = (x'_1 - x'_2)(y_1 - y_2) - (x_1 - x_2)(y'_1 - y'_2)$

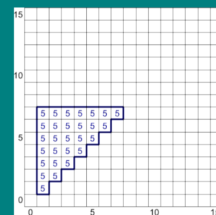


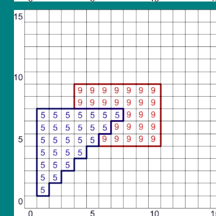


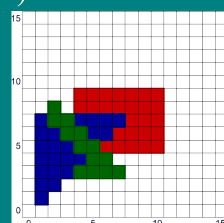


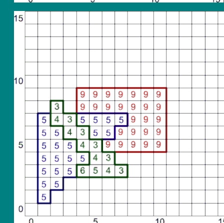
Z-Buffer

- First polygon
  - (1, 1, 5), (7, 7, 5), (1, 7, 5)
  - scan it in with depth
- Second polygon
  - (3, 5, 9), (10, 5, 9), (10, 9, 9), (3, 9, 9)
- Third polygon
  - (2, 6, 3), (2, 3, 8), (7, 3, 3)









Z-Buffer Algorithm

- Originally Cook, Carpenter, Catmull
- Given

  List of polygons $\{P_1, P_2, \ldots., P_n\}$

  An array x-buffer[x,y] initialized to +infinity

  An array Intensity[x,y]

- Begin

```
For each polygon P in selected list do
    For each pixel (x,y) that intersects P do
        Caluclate z-depth of P at (x,y)
        If z-depth < z-buffer[x,y] then
            Intensity[x,y] = intensity of P at (x,y)
            Z-buffer[x,y] = z-depth
Display Intensity array
```

Z-Buffer: Calculating Z-depth

- From plane equation, depth at position (x,y):

  $z = (-Ax - By - D)/C$

- Incrementally across scanline (x+1, y)

  $z' = (-A(x+1) - By - D)/C$

  $= (-Ax - By - D)/C - A/C$

  $= z - A/C$

- Incrementally between scanlines (x', y+1)

  $z' = (-A(x') - B(y+1) - D)/C$

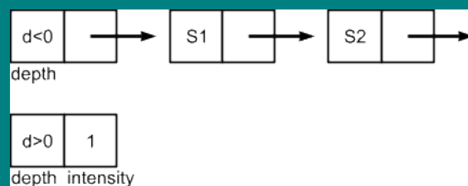
  $= z - (A/m + B)/C$

Z-Buffer Characteristics

- Good
  - Easy to implement
  - Requires no sortng of surfaces
  - Easy to put in hardware
- Bad
  - Requires lots of memory (about 9MB for 1280x1024 display)
  - Can alias badly (only one sample per pixel)
  - Cannot handle transparent surfaces

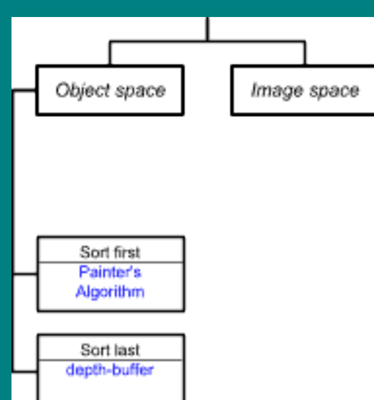
A-Buffer Method

- Basically z-buffer with additional memory to consider contribution of multiple surfaces to a pixel
- Need to store
  - Color (rgb triple)
  - Opacity
  - Depth
  - Percent area covered
  - Surface ID
  - Misc rendering parameters
  - Pointer to next



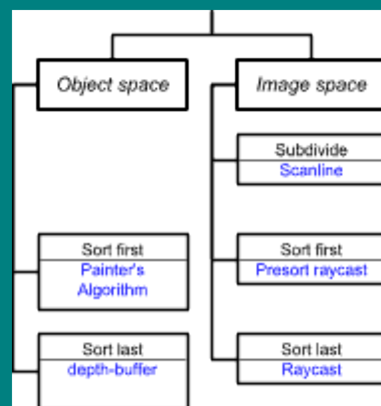
Taxonomy of Visibility Algorithms

- Ivan Sutherland -- A Characterization of Ten Hidden Surface Algorithms
- Basic design choices
  - Space for operations
    - Object
    - Image
  - Object space
    - Loop over objects
    - Decide the visibility of each
  - Timing of object sort
    - Sort-first
    - Sort-last



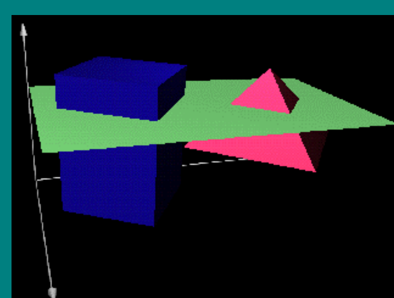
Taxonomy of Visibility Algorithms

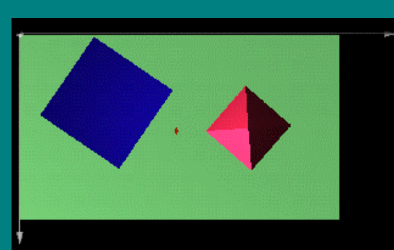
- Image space
  - Loop over pixels
  - Decide what's visible at each
- Timing of sort at pixel
  - Sort first
  - Sort last
  - Subdivide to simplify



Scanline Algorithm

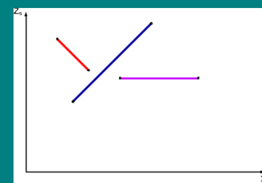
- Simply problem by considering only one scanline at a time
- intersection of 3D scene with plane through scanline



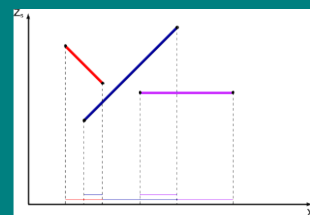


Scanline Algorithm

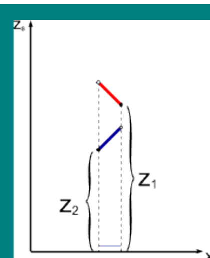
- Consider xz slice



- Calculate where visibility can change



- Decide visibility in each span

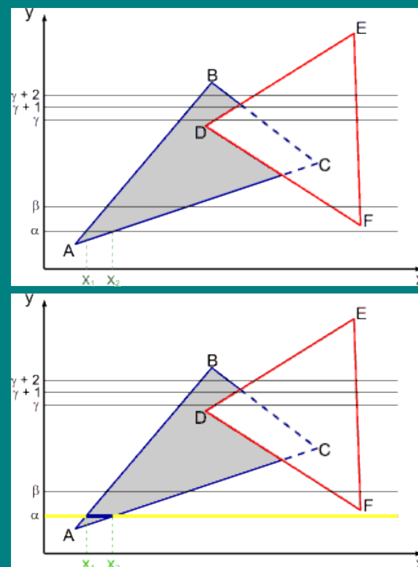


Scanline Algorithm

```
1.  Sort polygons into sorted surface table (SST)
    based on Y
2.  Initialize y and active surface table (AST)
   Y = first nonempty scanline
   AST = SST[y]
3.  Repeat until AST and SST are empty
   Identify spans for this scanline (sorted on x)
   For each span
      determine visible element (based on z)
      fill pixel intensities with values from element
   Update AST
      remove exhausted polygons
      y++
      update x intercepts
      resort AST on x
      add entering polygons
4.  Display Intensity array
```

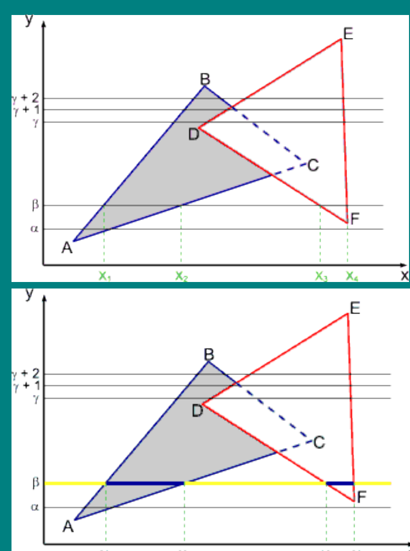
Scanline Visibility Algorithm

- Scanline α
  - AST: ABC
  - Spans
    - $0 \rightarrow x_1$ background
    - $x_1 \rightarrow x_2$ ABC
    - $x_2 \rightarrow max$ background



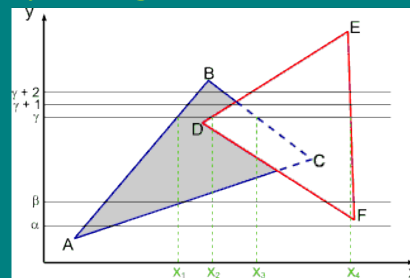
Scanline Visibility Algorithm

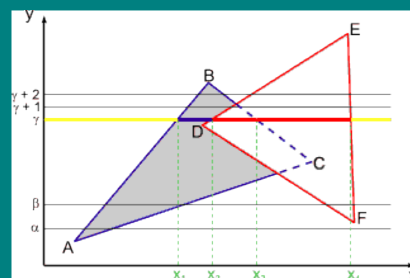
- Scanline β
  - AST: ABC DEF
  - Spans
    - $0 \rightarrow x_1$ background
    - $x_1 \rightarrow x_2$ ABC
    - $x_2 \rightarrow x_3$ background
    - $x_3 \rightarrow x_4$ DEF
    - $x_4 \rightarrow max$ background



Scanline Visibility Algorithm

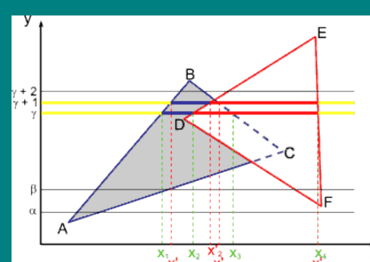
- Scanline γ
  - AST: ABC DEF
  - Spans
    - 0 -> x_1 background
    - x_1 -> x_2 ABC
    - x_2 -> x_3 DEF
    - x_3 -> x_4 DEF
    - x_4-> max background

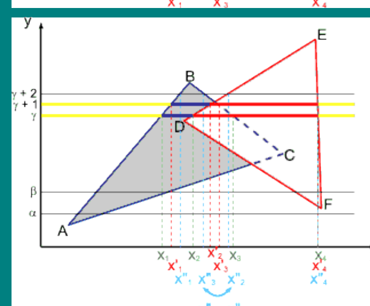




Scanline Visibility Algorithm

- Scanline γ + 1
  - Spans
    - 0 -> x_1 background
    - x_1 -> x_2 ABC
    - x_2 -> x_3 DEF
    - x_3 -> x_4 DEF
    - x_4-> max background
- Scanline γ + 2
  - Spans
    - 0 -> x_1 background
    - x_1 -> x_2 ABC
    - x_2 -> x_3 background
    - x_3 -> x_4 DEF
    - x_4-> max background



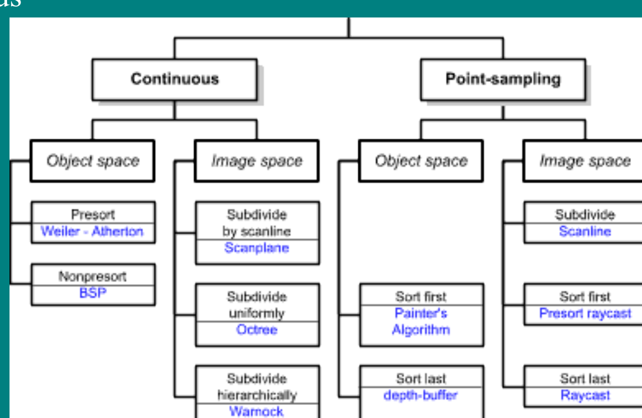


Characteristics of Scanline Algorithm

- Good
  - Little memory required
  - Can generate scanlines as required
  - Can antialias within scanline
  - Fast
    - Simplification of problem simplifies geometry
    - Can exploit coherence
- Bad
  - Fairly complicated to implement
  - Difficult to antialias between scanlines

Taxonomy Revisted

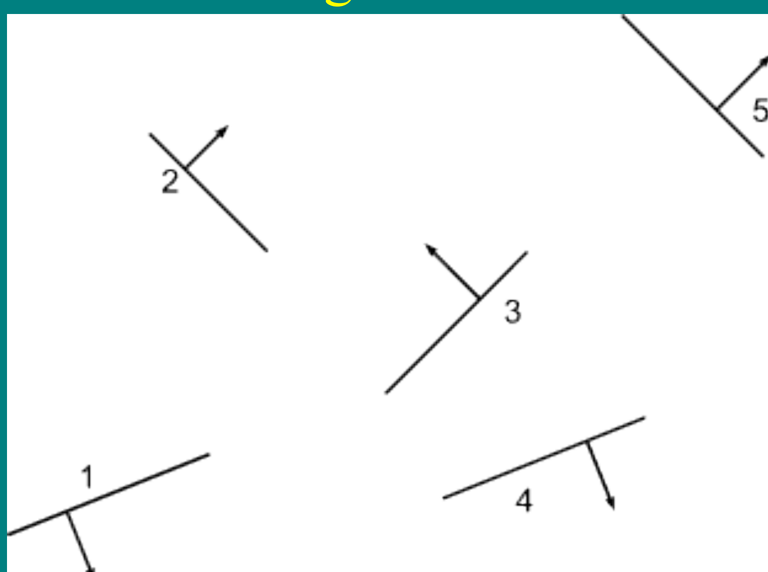
- Another dimension
  - Point-sampling
  - continuous



BSP Tree: Building the Tree

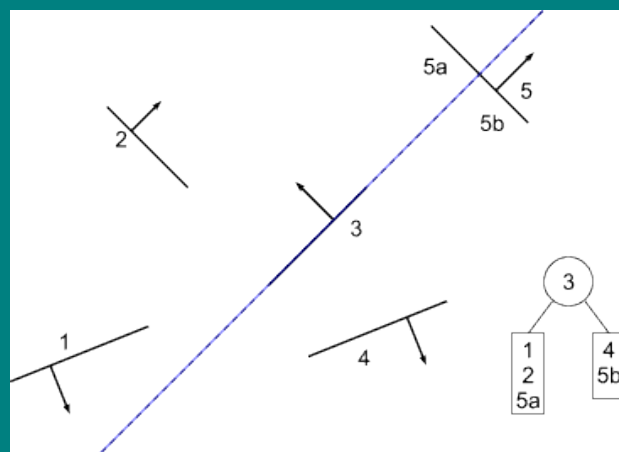
```
BSPTree MakeBSP ( Polygon list ) {
   if ( list is empty ) return null
   else {
      root = some polygon ; remove it from the list
      backlist = frontlist = null
      for ( each remaining polygon in the list ) {
         if ( p in front of root )
            addToList ( p, frontlist )
         else if ( p in back of root )
            addToList ( p, backlist )
         else {
            splitPolygon (p,root,frontpart,backpart)
            addToList ( frontpart, frontlist )
            addToList ( backpart, backlist )
            }
       }
   return (combineTree(MakeBSP(frontlist),root,
                              MakeBSP(backlist)))
   }
}
```

Building a BSP Tree



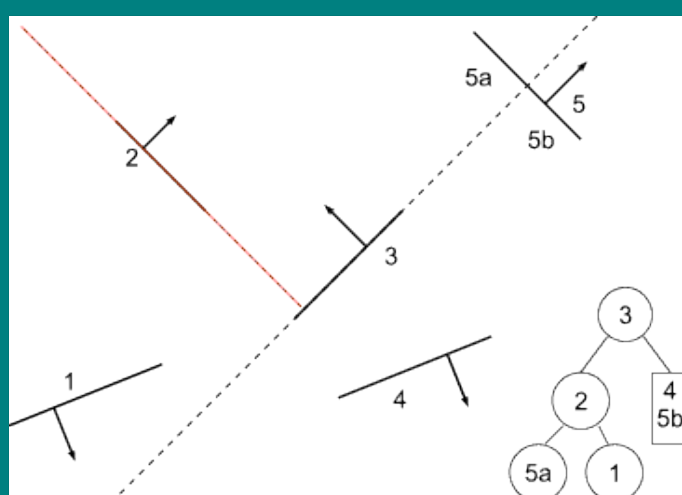
Building a BSP Tree

- Use pgon 3 as root, split on its plane
- Pgon 5 split into 5a and 5b



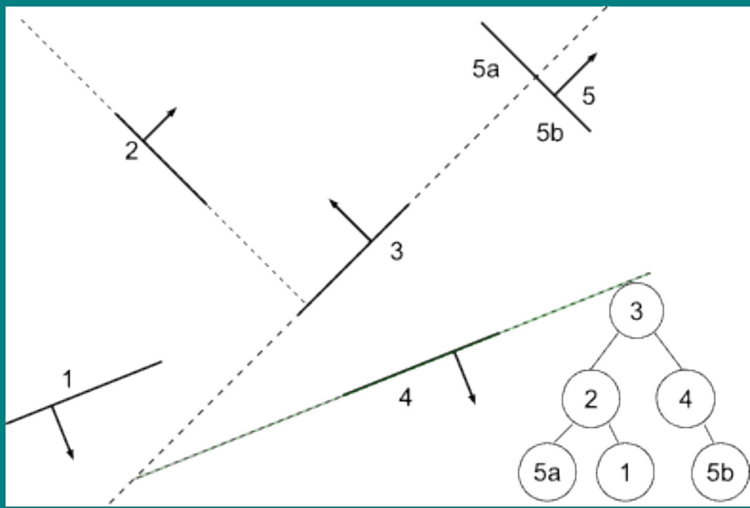
Building a BSP Tree

- Split left subtree at pgon 2



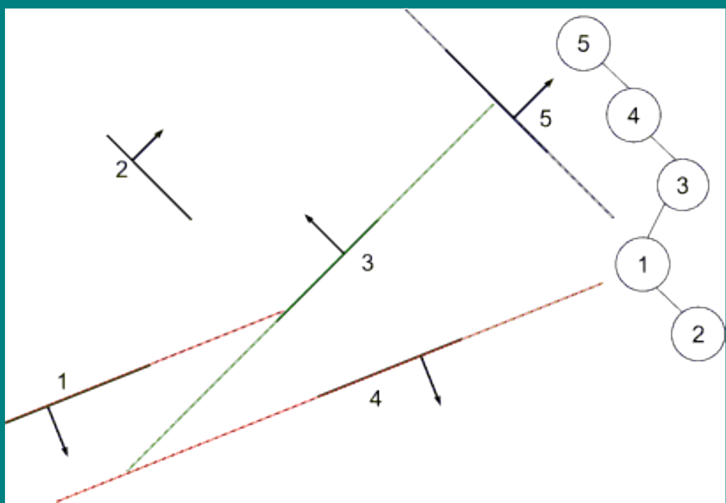
Building a BSP Tree

- Split right subtree at pgon 4



Building a BSP Tree

- Alternate tree if splits are made at 5, 4, 3, 1

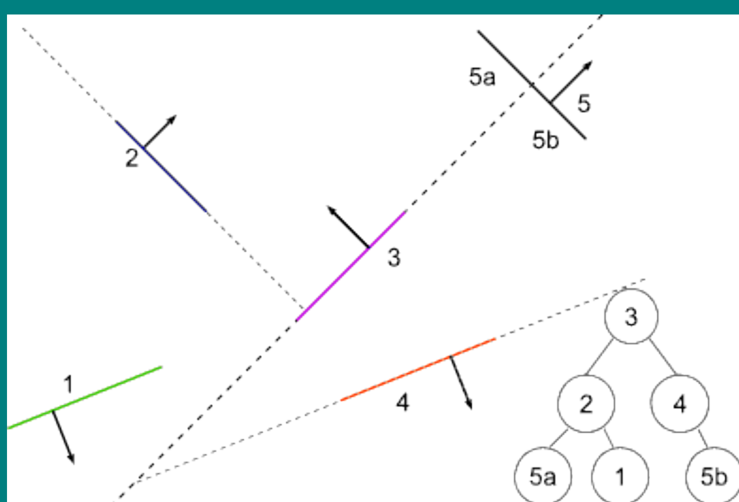


BSP Tree: Displaying the Tree

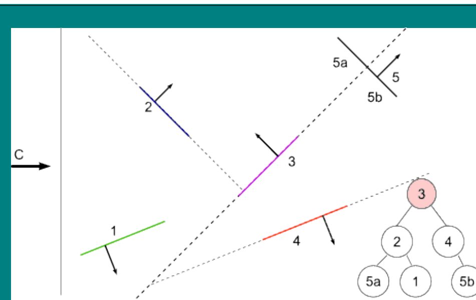
```
DisplayBSP ( tree )
{
   if ( tree not empty ) {
      if ( viewer in front of root ) {
         DisplayBSP ( tree -> back )
    DisplayPolygon ( tree -> root )
    DisplayBSP ( tree -> front )
      }
      else {
         DisplayBSP ( tree -> front )
    DisplayPolygon ( tree -> root )
    DisplayBSP ( tree -> back )
      }
   }
}
```

BSP Tree Display

- Built BSP tree structure



BSP Tree Display

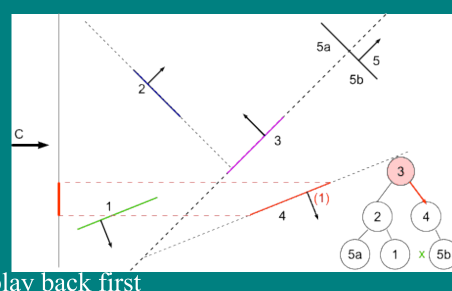


For view point at C

at 3 : viewpoint on front -> display back first

at 4 : viewpoint on back -> display front first

BSP Tree Display



For view point at C

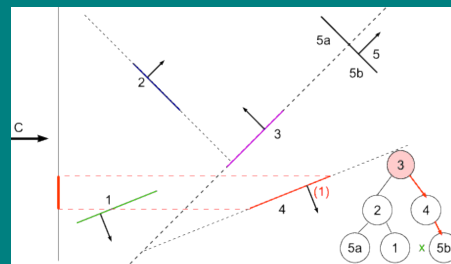
at 3 : viewpoint on front -> display back first

at 4 : viewpoint on back -> display front first (none)

display self

display back

BSP Tree Display



For view point at C

at 3 : viewpoint on front -> display back first

at 4 : viewpoint on back -> display front first (none)

display self

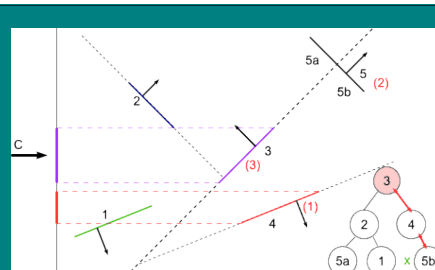
display back

at 5b : viewpoint on back -> display front (none)

display self

display back (none)

BSP Tree Display



For view point at C

at 3 : viewpoint on front -> display back first

at 4 : viewpoint on back -> display front first (none)

display self

display back

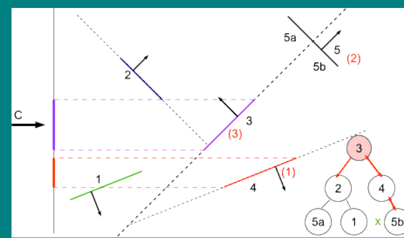
at 5b : viewpoint on back -> display front

display self

display back (none)

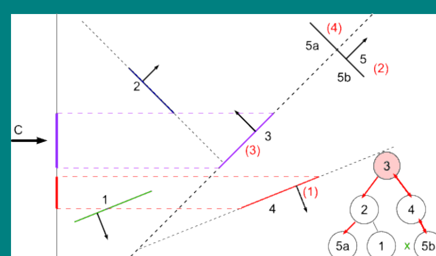
display self

BSP Tree Display



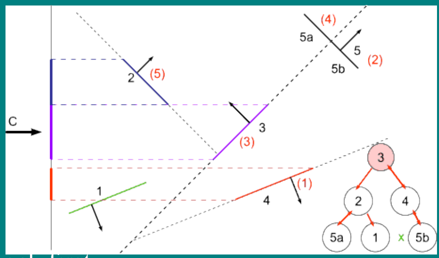
For view point at C
at 3 : viewpoint on front -> display back first
at 4 : viewpoint on back -> display front first (none)
display self
display back
at 5b : viewpoint on back -> display front
display self
display back (none)
display self
display front

BSP Tree Display



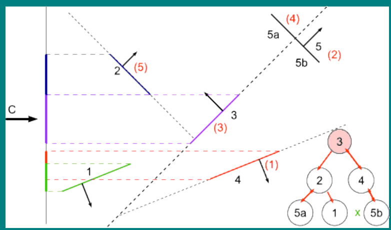
For view point at C
at 3 : viewpoint on front -> display back first
at 4 : viewpoint on back -> display front first (none)
display self
display back
at 5b : viewpoint on back -> display front
display self
display back (none)
display self
display front
at 2 : viewpoint on back -> display front first

BSP Tree Display



For view point at C
at 3 : viewpoint on front -> display back first
at 4 : viewpoint on back -> display front first (none)
display self
display back
at 5b : viewpoint on back -> display front
display self
display back (none)
display self
display front
at 2 : viewpoint on back -> display front first
at 5a : viewpoint on back -> display front (none)
display self
display back (none)

BSP Tree Display



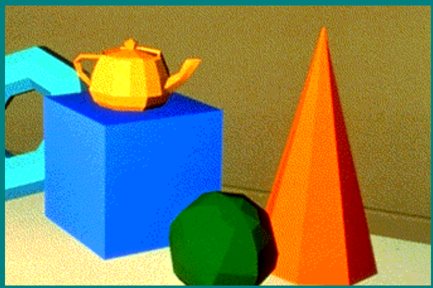
For view point at C
at 3 : viewpoint on front -> display back first
at 4 : viewpoint on back -> display front first (none)
display self
display back
at 5b : viewpoint on back -> display front
display self
display back (none)
display self
display front
at 2 : viewpoint on back -> display front first
at 5a : viewpoint on back -> display front (none)
display self
display back (none)
display self
at 1 : viewpoint on back -> display front (none)
display self
display back (none)

Shading Revisited

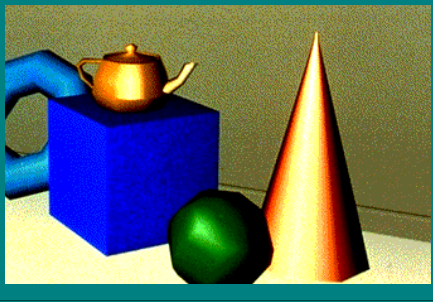
- Illumination models compute appearance at a location
- How do you efficiently fill areas?

Diffuse Shading Models

Flat shading



Gouraud shading



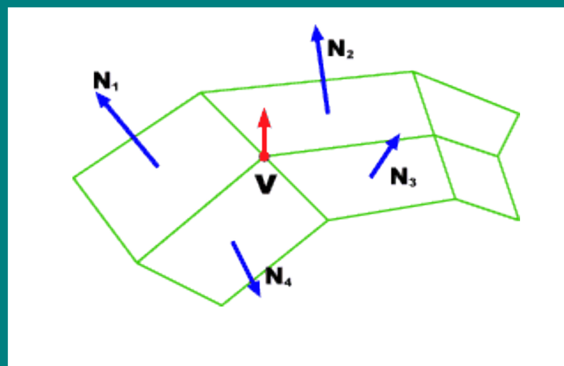
Flat Shading Algorithm

```
For each visible polygon
   Evaluate illumination with polygon
      normal
   For each scanline
      For each pixel on scanline
         Fill with calculated intensity
```

Interpolated Shading Algorithm

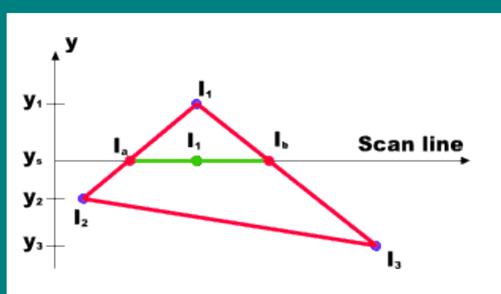
```
For each visible polygon
   For each vertex
     Evaluate illumination with vertex
      normals
   For each scanline
      Interpolate intensity along edges
            (for span extrema)
      For each pixel on scanline
         Interpolate intensity from
            extrema
```

Vertex Normals



- The normal vector at vertex V is calculated as the average of the surface normals for each polygon sharing that vertex

Gouraud Calculations



$I_a = I_1+(I_2-I_1)/(y_a-y_1)/(y_2-y_1)$

$I_b = I_1+(I_3-I_1)/(y_b-y_1)/(y_3-y_1)$

$I_p = I_a+(I_b-I_a)/(x_p-x_a)/(x_b-x_a)$

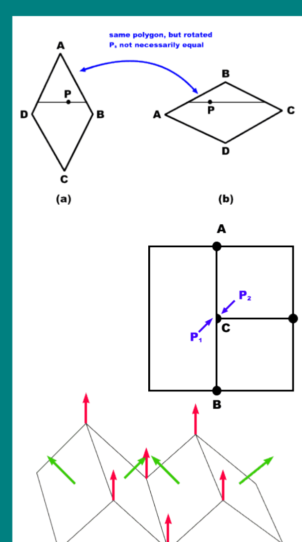
1. Calculate intensity at vertices (I_1, I_2, I_3)
2. Interpolate vertex intensities along edges (I_a, I_b)
3. Interpolate intensities at span extrema to pixels (I_p)

Barycentric Rasterization

```
xmin = floor(xi)
xmax = ceiling(xi)
ymin = floor(yi)
ymax = ceiling(yi)
for y = ymin to ymax do
    for x = xmin to xmax do
        α = f12(x,y)/f12(x0,y0)
        β = f20(x,y)/f20(x1,y1)
        γ = f01(x,y)/f01(x2,y2)
        If (α ∈ [0,1] and β ∈ [0,1] and γ ∈ [0,1] then
            c0 = evaluate_illumination(x0,y0,z0)
            c1 = evaluate_illumination(x1,y1,z1)
            c2 = evaluate_illumination(x2,y2,z2)
            c = αc0 + βc1 + γc2
            Draw pixel (x,y) with color c
```

Problems with Interpolated Shading

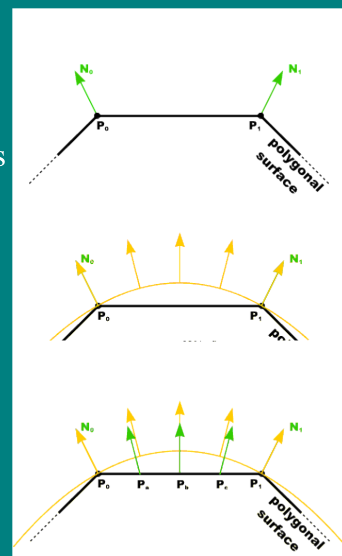
- Polygon silhouette
- Perspective distortion
- Orientation dependence
- Problems at shared vertices
- Unrepresentative vertex normals



Phong Shading

- Ideally: shade from normals of curved surface
- Approximate with normals interpolated between vertex normals

$N_a = |P_a - P_0| / |P_1 - P_0| N_1 + |P_1 - P_a| / |P_1 - P_0| N_0$



Phong Algorithm

- For each visible polygon
  - For each scanline
    - Calculate normals at edge intersections (span extrema) by linear interpolation
    - For each pixel on scanline
      - Calculate normal by interpolation of normals at span extrema
      - Evaluate illumination model with that normal

Barycentric Rasterization

```
x_min = floor(x_i)
x_max = ceiling(x_i)
y_min = floor(y_i)
y_max = ceiling(y_i)
for y = y_min to y_max do
   for x = x_min to x_max do
      α = f12(x,y)/f12(x0,y0)
      β = f20(x,y)/f20(x1,y1)
      γ = f01(x,y)/f01(x2,y2)
      If (α ∈ [0,1] and β ∈ [0,1] and γ ∈ [0,1] then
         n = αn0 + βn1 + γn2
         Normalize (n)
         c = evaluate_illumination(x,y,n)
         Draw pixel (x,y) with color c
```

Artistic Illumination

- Concept: intentionally mimic artistic effects which may not match photorealism (NPR)
- Examples
  - Line drawing
  - Shading effects
    - Cool-warm (tone shading)
    - Toon
  - Media Emulation

Silhouette Drawing

- Want to draw silhouette edge to emphasize shape
- Silhouette defined by points where surface normal is orthogonal to view vector

  $V \bullet N = 0$
- Implementation for polygonal meshes: draw edge when pgons change from forward to back

```
if (V•N0)(V•N1) ≤ 0
    Draw silhouette (edge between pgons)
```

- Add sharp creases

```
if (N0•N1) ≤ threshold
    Draw silhouette (edge between pgons)
```

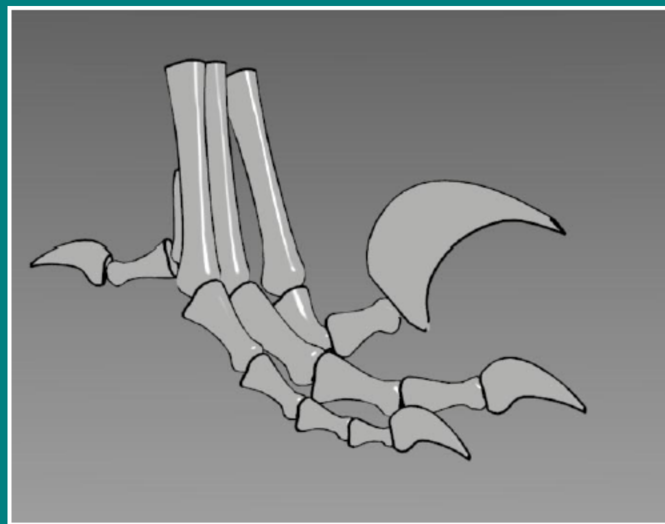
Diffuse Only

Kd = 1, ka=0

Gooch '98

Highlights and Edges

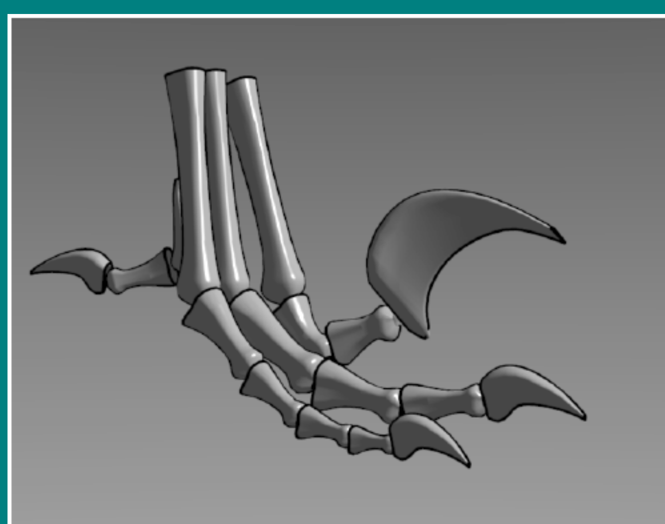
Gooch 98



Phong Shading and Edges

Kd=.5
Ka = .1

Gooch 98



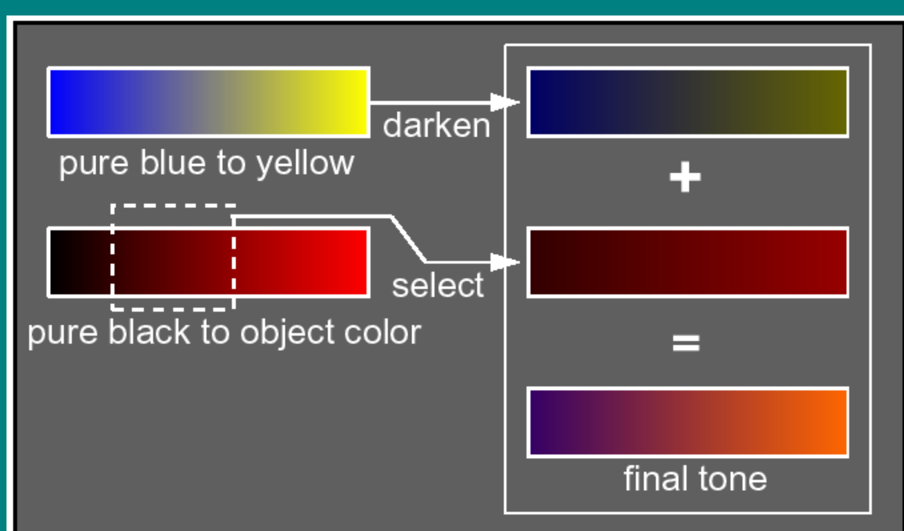
Tone Shading Model

$$I = \left(\frac{1+\hat{\mathbf{l}}\cdot\hat{\mathbf{n}}}{2}\right) k_{cool} + \left(1 - \frac{1+\hat{\mathbf{l}}\cdot\hat{\mathbf{n}}}{2}\right) k_{warm}$$

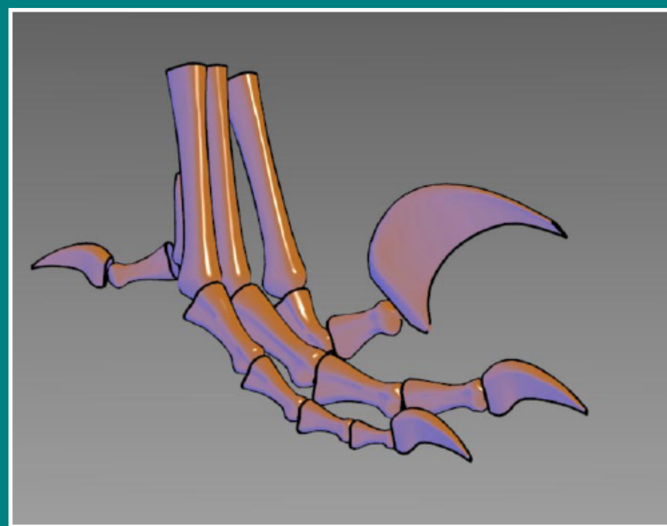
with

$$\begin{aligned} k_{cool} &= k_{blue} + \alpha k_d \\ k_{warm} &= k_{yellow} + \beta k_d \end{aligned}$$

Mixing Tone and Color



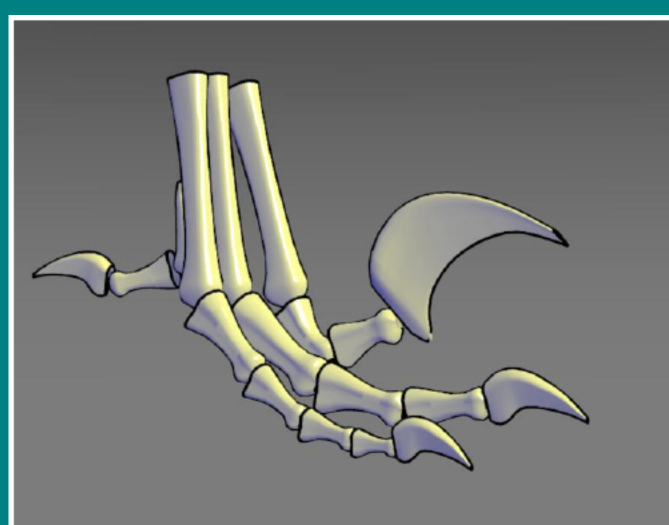
Constant Luminance Tone



Gooch 98

Luminance/Tone Rendering

B=0.4, y=0.4
α=.2, β = .6

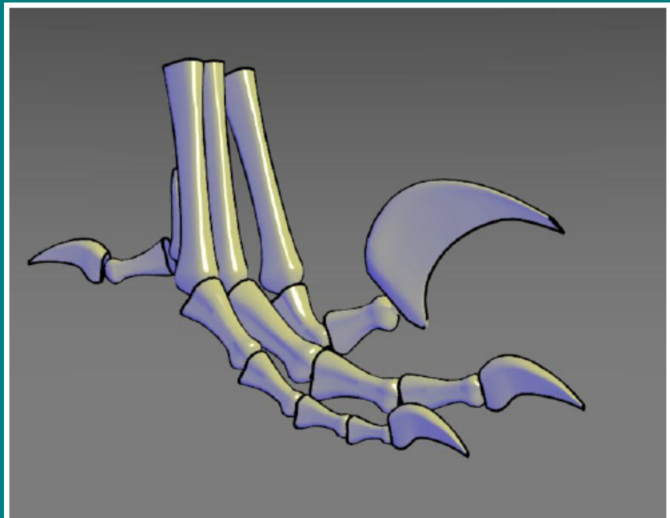


Gooch 98

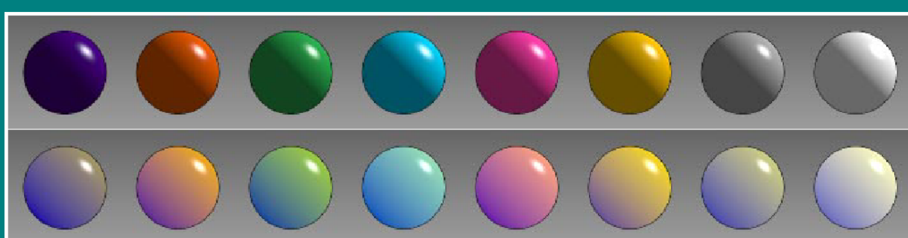
Luminance/Tone Rendering

B=0.55, y=0.8
α = .25, β=.5

Gooch 98

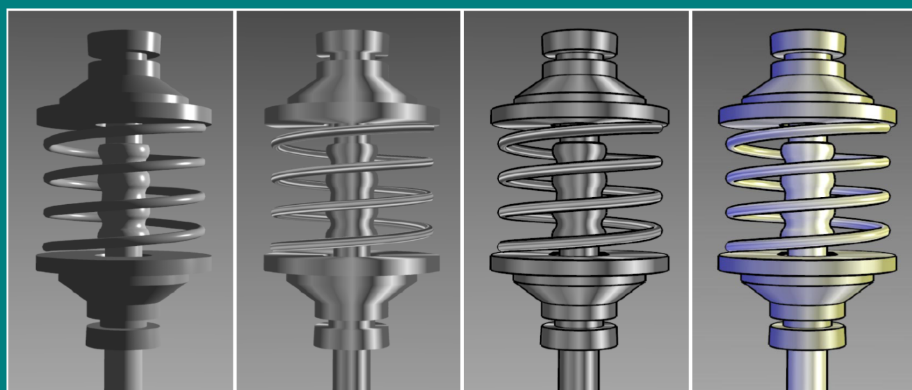


Hue/Tone Interactions



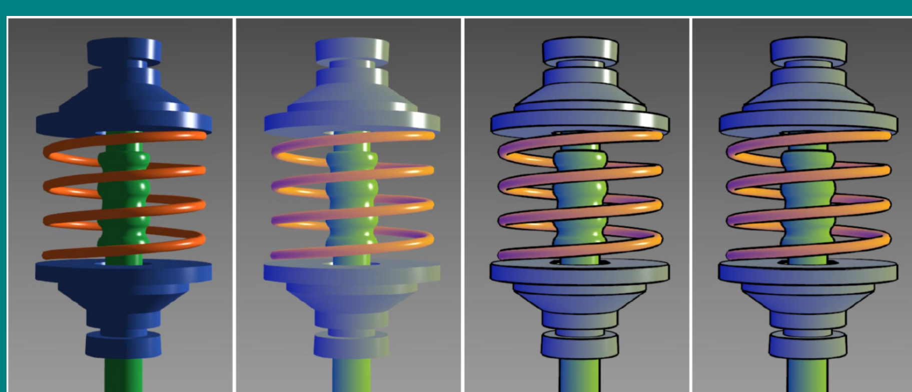
- Gooch 98

Tone/Metal



Gooch 98

Tone/Color



Gooch 98