CMSC 435
Introductory Computer Graphics
Math Review

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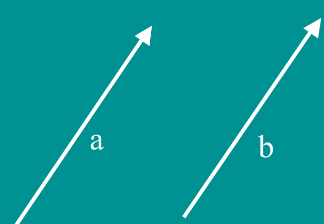
UMBC

Vectors

- Vector: length and direction gives offset

$$a = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} \qquad a^T = \begin{bmatrix} x_a & y_a & z_a \end{bmatrix}$$

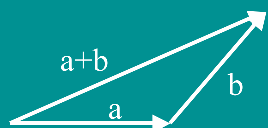


Vectors (2)

- Operations
  - Length

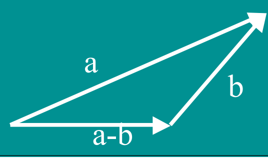
$$\|a\| = \sqrt{x_a^2 + y_a^2 + z_a^2}$$

  - Addition



$$a + b = \begin{bmatrix} x_a + x_b \\ y_a + y_b \\ z_a + z_b \end{bmatrix}$$

  - Subtraction



$$a - b = \begin{bmatrix} x_a - x_b \\ y_a - y_b \\ z_a - z_b \end{bmatrix}$$

Vectors (3)

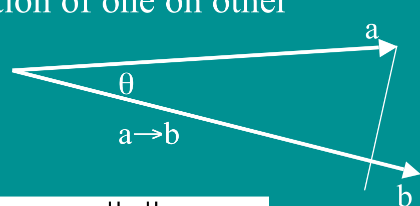
- Dot Product

$$a \cdot b = x_a x_b + y_a y_b + z_a z_b$$

  - angle between

$$a \cdot b = \|a\|\|b\|\cos\theta$$

  - projection of one on other



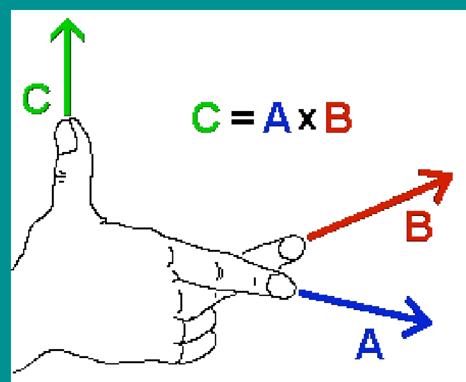
$$a \rightarrow b = \|a\|\cos\theta$$

Vectors (4)

- Cross Product

$$a \times b = (y_a z_b - z_z y_b, z_a x_b - x_a z_b, x_a y_b - y_a x_b)$$

- c= a×b
  - orthogonal to a,b
  - length=pgram area
- b×a = -a×b



Vectors (5)

- Coordinate Frames (uvw coord system)
  - Orthonormal basis
    - Unit length $\|u\| = \|v\| = \|w\| = 1$
    - Orthogonal $u \cdot v = v \cdot w = w \cdot u = 0$
  - Right-handed vs left-handed
    - Right-handed $w = u \times v$
- Coordinate frames
  - Global (world) coordinate system
  - Local (object) coordinate system

Linear Interpolation

- Formula

$$n = n_1 + t(n_2 - n_1)$$

where $n=n_1$ at $t=0$, $n=n_2$ at $t=1$

- Uses
  - Points along a line (repeat for x,y,z)
  - Colors between sample points (repeat for r,g,b)

Implicit Lines

- Function defines relationship among coords
- 2D Formula

$$y - mx - b = 0$$

  - where y is slope and b is intercept
- General form

$$Ax + By + C = 0$$

  - Orthogonal to $\begin{bmatrix} A & B \end{bmatrix}$
  - Computing from points

$$(y_0 - y_1)x + (x_0 - x_1)y + x_0 y_1 - x_1 y_0 = 0$$

  - Substitute in point for signed distance

Implicit Planes

- Function defines relationship among coords
- General form

$$(\mathrm{p}-\mathrm{a})\cdot\mathrm{n}=0$$

  - Orthogonal to n, through a
  - Computing from points a,b,c

$$(y_0-y_1)x+(p-a)\cdot((b-a)\times(c-a))=0$$

Parametric Lines

- Define line by end points
- Formula

$$p=p_1+t(p_2-p_1)$$

where $p=p_1$ at $t=0$, $p=p_2$ at $t=1$

- Components

$$x=x_1+t(x_2-x_1)$$

$$y=y_1+t(y_2-y_1)$$

$$z=z_1+t(z_2-z_1)$$

Matrix Multiplication

- With matrices A, B

$$A = \begin{bmatrix} a_{00} & a_{10} & a_{20} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{00} & b_{10} & b_{20} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix}$$

- To compute C=AB

$$c_{ij} = a_{i0}b_{0j} + a_{i1}b_{1j} + a_{i2}b_{2j}$$

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- To compute C=AB

$$c_{ij} = a_{i0}b_{0j} + a_{i1}b_{1j} + a_{i2}b_{2j}$$

- For example

$$c_{10} = a_{10}b_{00} + a_{11}b_{10} + a_{12}b_{20}$$