

CMSC 442/653
Instructor: Dr. Lomonaco
Homework 5

- **Listening Assignment:** Listen to George Gershwin's Rhapsody in Blue, <https://www.youtube.com/watch?v=eFHdRkeEnpM>
- **Reading Assignment:** <https://www.csee.umbc.edu/~lomonaco/f14/653/handouts/EU-Alg.pdf>
- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapters 2, 3, 6.

1) Let V be the Extended Hamming $[16,11]d=4$ binary linear code.

- a) Write down the parity check matrix H .
- b) Assuming the Binary Erasure Channel (BEC), if

$$\vec{r} = 10?1 \quad 1001 \quad ?001 \quad 1001$$

is the received vector, then what is the most likely most likely codeword that was originally sent? Please explain how you obtained your answers.

2) Let V be the above $[16,11]d=4$ Extended Hamming code.

- a) Using the above parity check matrix H , construct a maximal likelihood Error/Syndrome table for V .

b) Consider the following decoding procedure for V :

- i) If the syndrome is $\mathbf{0} = 00000$, then assume no error has occurred.
- ii) If the overall parity check is $\mathbf{1}$, then assume that a single error has occurred, and use the remaining three bits of the syndrome to correct the error.
- iii) If the overall parity check symbol is $\mathbf{0}$, but at least one of the other syndrome bits fail, then assume that an undetectable error has occurred.

Is this a best maximal likelihood decoding procedure? Explain why or why not.

3) Let V be a binary code of length $n=9$ capable of detecting error patterns of two or fewer bits, but incapable of detecting all errors of three or more bits. If a code vector $v \in V$ is sent over a BSC (Binary Symmetric Channel) with transition probability (i.e., with probability of error) $p=1/10$, then what is the probability P_U of an undetectable error? Please explain your answer.

4) Use the Extended Euclidean Algorithm to compute the multiplicative inverse

$$497^{-1} \pmod{899}$$

Please show your work.

5) Use the Extended Euclidean Algorithm to compute the multiplicative inverse

$$(x^5 + x^2 + 1)^{-1} \pmod{x^{10} + x^3 + 1}$$

Please your work.