

# CMSC 441

## Homework 3

### Reading Assignment:

- Listen to [Camille Saint-Saens' Danse Macabre](#).
- Read Chapter 4 Section 5 of text, and read the Brassard/Bratley (BB) [handout](#)

### Homework:

- 1) Problem 2.3.7, page 75 of BB handout
- 2) Problem 2.3.9, page 76 of BB handout
- 3) Problem 2.3.10, page 76 of BB handout
- 4) Problem 2.3.12, page 76 of BB handout
- 5) Exercise 4.5-1, page 96 of text
- 6) The  $n$ -th Fibonacci number  $F(n)$  is defined by the following recursion

$$\begin{cases} F(n) = F(n-1) + F(n-2) & \text{for } n \geq 2 \\ F(0) = 0, \quad F(1) = 1 \end{cases}$$

Given that the Fibonacci numbers satisfy the following equality

$$\begin{bmatrix} F(n-1) & F(n) \\ F(n) & F(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n \quad \text{for } n \geq 2,$$

construct (in pseudo code) an algorithm that computes the  $n$ -th Fibonacci number in time complexity  $\Theta(\log n)$ . Then explain why your algorithm is of time complexity  $\Theta(\log n)$ .

**Hint.** Use the method of repeated squares for computing matrix powers.

- 7) Determine the asymptotic time efficiency of the following algorithm:

```
Algorithm GE (  $A[0..n-1,0..n]$  )
//Input: An  $n \times (n+1)$  matrix  $A[0..n-1,0..n]$  of reals
for  $i \leftarrow 0$  to  $n-2$  do
  for  $j \leftarrow i+1$  to  $n-1$  do
    for  $k \leftarrow i$  to  $n$  do
       $A[j,k] \leftarrow A[j,k] - A[i,k]*A[j,i]/A[i,i]$ 
```

Be sure to explain how you got your answer.