CMSC 441
Homework 2

Reading Assignment:
- Listen to Igor Stravinsky's Firebird
- Read Chapters 1 through 3 of text

Homework:

1) There are four people who want to cross a rickety bridge; they all begin on the same side. You have 17 minutes to get them across to the other side. It's night, and they have one flashlight. A maximum of two people can cross the bridge at one time. Any party that crosses, either one or two people, must have a flashlight with them. The flashlight must be walked back and forth; it cannot be thrown. Person 1 takes 1 minute to cross the bridge, person 2 takes 2 minutes, person 3 takes 5 minutes, and person 4 takes 10 minutes. A pair must walk together at the rate of the slower person's pace.

2) Place a single grain of wheat on the first square of a chessboard, two on the second, four on the third, eight on the fourth, sixteen on the fifth, and so on, until all 64 squares had been filled. If it takes just 1 second to count each rain, how long illd it take to count all the grain on the chessboard?

3) For each of the following functions, indicate the tightest class $O(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Justify your assertions.

   a) $\left(n^2 + 1\right)^{10}$

   b) $\sqrt{10n^2 + 7n + 3}$

   c) $2n \lg^2 (n+2) + (n+2)^2 \lg\left(\dfrac{n}{2}\right)$

   d) $2^{n+1} + 3^{n-1}$

   e) $\left\lfloor \log_2\left(n^{100}\right) \right\rfloor$

4) Compute the following sums

   a) $\sum_{i=1}^{n} \sum_{j=1}^{n} ij$

   b) $\sum_{i=1}^{n} \dfrac{1}{i(i+1)}$

5) Consider the following algorithm:

Algorithm *Enigma* (**A[0..n-1,0..n-1]**)
   //Input: A matrix **A[0..n-1,0..n-1]** of real numbers
   for i ← 0 **to** n-2 **do**
      for j ← i+1 **to** n-1 **do**
         if A[i,j] ≠ A[j,i] **return** "false"
   return "true"

a) What does the algorithm compute?
b) What is the basic (i.e., dominant) operation
c) What is the time complexity of this algorithm expressed in asymptotic notation?
d) Can you improve the algorithm?