CMSC 442/653
Spring 2009
Instructor: Dr. Lomonaco
Homework 7

- Listen to Brahms' Symphony No. 1
- **Reading Assignment:** Review relevant slides on "Overview of Coding Theory" found at http://www.cs.umbc.edu/~lomonaco/f06/653/Slides653.html
- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapters 6 &8.

1UG) Let V be the Hamming $[15,11]\ d=3$ binary linear code.

a) Write down the parity check matrix H.

b) If

$$\vec{r} = 1000 \quad 1000 \quad 0000 \quad 001$$

is a received vector, then what is the most likely error pattern. What is the most likely codevector that was originally sent? Please explain how you obtained your answers.

2UG) Construct the addition and multiplication tables for the ring

$$R_4 = GF(2)[x]/\left(x^4+1\right)$$

3UG) Let V be the cyclic code in $R_{15} = GF(2)[x]/\left(x^{15}+1\right)$ given by the generator polynomial

$$g(x) = x^8 + x^4 + x^2 + x + 1 \ .$$

a) What is the length n of V?

b) What is the dimension k of V?

c) Use the generator polynomial $g(x)$ to construct a generator matrix G for V.

d) What is the parity check polynomial $h(x)$ of V ?

e) Use the parity check polynomial $h(x)$ to construct the parity check matrix H of V .

4UG) Given that

$$x^9 + 1 = (x+1)\left(x^2+x+1\right)\left(x^6+x^3+1\right)$$

is a complete factorization over $GF(2)$ of x^9+1 into irreducible polynomials,

a) Draw the lattice of all ideals in $R_9 = GF(2)[x]/\left(x^9+1\right)$.

b) Determine the dimension of each ideal in R_9.

c) Determine the number of elements in each ideal in R_9.

d) List all the elements of the ideals

$$\left(x^6 + x^3 + 1\right) \text{ and } \left((x+1)(x^6 + x^3 + 1)\right)$$

5G) Prove that, if a binary linear code V has at least one vector of odd Hamming weight, then half the code vectors are of even Hamming weight, and the other half are of odd Hamming weight. You may assume without proof the following proposition:

Proposition. *Let V be a binary linear code, and let $H : V \to \mathbb{N}$ denote the Hamming weight function. If u and v are two arbitrary vectors in V, then*

a) $H\left((u+v)\right)$ is even iff either both $H(u)$ and $H(v)$ are even or both $H(u)$ and $H(v)$ are odd.

b) $H\left((u+v)\right)$ is odd iff either $H(u)$ is even and $H(v)$ is odd or $H(u)$ is odd and $H(v)$ is even.