CMSC 442/653
Spring 2009
Instructor: Dr. Lomonaco

Homework 3

- **Listening Assignment:** Listen to Rachmanioff's Piano Concerto No. 4
- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapters 2, 3, 6.
- **Read**

http://www.csee.umbc.edu/~lomonaco/f06/653/handouts/Peterson-Pages22-25.pdf

1UG) Consider the following degree 4 irreducible polynomial **p(x)** given in Peterson's Table of Irreducible Polynomials over **GF(2)**

DEGREE 4 ... 3 37D ...

a) Write down **p(x)** .

b) Since p(x) is irreducible and of degree 3, it follows that

$$\mathbf{GF(2^4) = GF(2)[x] \bmod p(x)}$$

List all the elements of **GF(2^4)** in the above representation, i.e., in terms of

$$\xi = x \bmod p(x)$$

c) Let $\xi = x \bmod p(x)$. Why is $\left\{\xi^k\right\}$ not a complete list of all the non-zero elements of **GF(2^4)** ?

2UG) Consider the following matrix over **GF(2)**

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

a) Prove that the rows of **M** are linearly dependent.

b) Prove that the first three rows **M** form a basis for the row space of **M**.

c) What is the dimension of the row space of **M**? Explain your answer.

3UG) Consider the following matrix **S** over **GF(3)**

$$S = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{2} & \mathbf{0} & \mathbf{2} \\ \mathbf{2} & \mathbf{2} & \mathbf{0} & \mathbf{2} & \mathbf{1} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{2} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{1} \end{pmatrix}$$

a) Put the matrix **S** into echelon canonical form. (**Hint.** See section 2.6 of optional text)

b) Use the resulting echelon canonical form to find a basis for the row space of **S** . Explain your answer.

c) What is the dimension of the row space of **S** ? Explain how you found your answer.