CMSC 457/643 RESEARCH PROJECT ON QUANTUM ENTANGLEMENT

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1. Instructions

Write a research report on the project described in the following sections of this paper. Using the following format for your report:

Part 1. Executive summary of your report. This should be no more than one page, and should also include the project title, your name, and date.
Part 2. An explanation of what you have learned from this project. This section should be no more than 8 pages.
Part 3. A list of references you have used for the project
Part 4. A listing of your mathematica code

2. Research Questions to Be Answered

Consider the following entangled quantum states:

$$\begin{cases} |EPR\rangle_{ij} & = & \frac{|0_i 1_j\rangle - |1_i 0_j\rangle}{\sqrt{2}} \\ |GHZ\rangle_{ijk} & = & \frac{|0_i 0_j 0_k\rangle + |1_i 1_j 1_k\rangle}{\sqrt{2}} \\ |Werner\rangle_{ijk} & = & \frac{|0_i 0_j 1_k\rangle + |0_i 1_j 0_k\rangle + |1_i 0_j 0_k\rangle}{\sqrt{3}} \end{cases}$$

Question 1. What state results if a Bell measurment is made on qubits 1 and 2 of the following state

$$|EPR\rangle_{01} |EPR\rangle_{23} \ ?$$

Question 2 What state results if a Bell measurment is made on qubits 2 and 3 of the following state

$$|GHZ\rangle_{012} |GHZ\rangle_{345} \ ?$$

Question 3. What state results, if after the Bell measurement executed in Question 2, a Bell measurment is made on the resulting state on qubits 0 and 5?

Date: November 10, 2010.

Qestion 4. What state results if a Bell measurment is made on qubits 2 and 3 of the following state

$$\left|Werner\right\rangle_{012} \left|GHZ\right\rangle_{345} \text{ ?}$$

Question 5. What state results if a Bell measurment is made on qubits 2 and 3 of the following state

$$\left|Werner\right\rangle_{012} \left|EPR\right\rangle_{34} \text{ ?}$$

Question 6. Free form question:

a) Give examples of Bell measurments on other possible states and combinations of states.

b) Do you see a pattern? What is it?

3. Suggested Mathematica Functions for Your Project

You will find below a list of suggested Mathematia functions that you may need to create to help you investigate the above questions on quantum entaglement:

- SpectralDecomposition[Ω]
  Input: Observable Ω
  Output: $\{\{\lambda_1, \lambda_2, \ldots, \lambda_m\}, \{P_1, P_2, \ldots, P_m\}\}$, where $\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$ is a complete set of the distinct eigenvalues of Ω, and where P_j is the projector corresponding to the eigenvalue λ_j.

- BellMeasurement[Ψ, i, j]
  Input: State ket Ψ and locations i and j of qubits to be measured
  Output: $\{\{\Psi_1, \Psi_2, \Psi_3, \Psi_4\}, \{p_1, p_2, p_3, p_4\}\}$, where p_k is the probabilty that state ket Ψ_k will be the result of the measurement.

- BellMeasurementRandomOutput[Ψ, i, j]
  Input: State ket Ψ and locations i and j of qubits to be measured
  Output: Output is state Ψ_j with probability p_j. [You will need to use the Mathematica randum number generator for his.]

- Measurement[Ψ, Ω]
  Input: State ket Ψ and observable Ω
  Output: $\{\{\lambda_1, \lambda_2, \ldots, \lambda_m\}, \{\Psi_1, \Psi_2, \ldots, \Psi_m\}, \{p_1, p_2, \ldots, p_m\}\}$, where the λ_j's are the distinct eigenvalues of Ω, where Ψ_j and p_j are respectively the coresponding state and probability.

- MeasurementRandomOutput[Ψ, Ω]
  Input: State ket Ψ and observable Ω
  Output: $\{\lambda_j, \Psi_j\}$ with probability p_j . [You will need to use the Mathematica randum number generator for his.]