

Answers to Homework 1.5

Example:

Measure $\text{ket}\psi = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{i}{\sqrt{3}} \\ 0 \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$		w.r.t. the observable $\Omega = \begin{pmatrix} 0 & 0 & 1 & -i \\ 0 & 0 & i & -1 \\ 1 & -i & 0 & 0 \\ i & -1 & 0 & 0 \end{pmatrix}$			
Eigenvalue	Basis	Projector	Prob	Resulting State	
$\sqrt{2}$	$\begin{pmatrix} -\frac{i}{2} \\ -\frac{1}{2} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ \frac{i}{2} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} \\ 0 & \frac{1}{2} & \frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & \frac{1}{2} & 0 \\ \frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} \frac{i+\sqrt{2}}{2\sqrt{3}} \\ \frac{2i+\sqrt{2}}{2\sqrt{6}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} \end{pmatrix}$	
$-\sqrt{2}$	$\begin{pmatrix} \frac{i}{2} \\ \frac{1}{2} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} \\ -\frac{i}{2} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2\sqrt{2}} & \frac{i}{2\sqrt{2}} \\ 0 & \frac{1}{2} & -\frac{i}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{i}{2\sqrt{2}} & \frac{1}{2} & 0 \\ -\frac{i}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & \frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} \frac{-i+\sqrt{2}}{2\sqrt{3}} \\ -\frac{2i+\sqrt{2}}{2\sqrt{6}} \\ -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} \end{pmatrix}$	

Exercise 1.1:

Measure $\text{ket}\psi = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$		w.r.t. the observable $\Omega = \begin{pmatrix} 2 & 0 & 0 & i \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ -i & 0 & 0 & 2 \end{pmatrix}$		
Eigenvalue	Basis	Projector	Prob	Resulting State
1	$\begin{pmatrix} -\frac{i}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} \frac{1}{2} - \frac{i}{2} \\ 0 \\ 0 \\ \frac{1}{2} + \frac{i}{2} \end{pmatrix}$
2	$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$
3	$\begin{pmatrix} \frac{i}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} \frac{1}{2} + \frac{i}{2} \\ 0 \\ 0 \\ \frac{1}{2} - \frac{i}{2} \end{pmatrix}$

Exercise 1.2:

Measure $\text{ket}\psi = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$		w.r.t. the observable $\Omega = \begin{pmatrix} \frac{5}{2} & 0 & 0 & \frac{3i}{2} \\ 0 & \frac{5}{2} & \frac{i}{2} & 0 \\ 0 & -\frac{i}{2} & \frac{5}{2} & 0 \\ -\frac{3i}{2} & 0 & 0 & \frac{5}{2} \end{pmatrix}$		
Eigenvalue	Basis	Projector	Prob	Resulting State
1	$\begin{pmatrix} -\frac{i}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} \frac{1}{2} - \frac{i}{2} \\ 0 \\ 0 \\ \frac{1}{2} + \frac{i}{2} \end{pmatrix}$
2	$\begin{pmatrix} 0 \\ -\frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{i}{2} & 0 \\ 0 & \frac{i}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ \frac{1}{2} - \frac{i}{2} \\ \frac{1}{2} + \frac{i}{2} \\ 0 \end{pmatrix}$
3	$\begin{pmatrix} 0 \\ \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{i}{2} & 0 \\ 0 & -\frac{i}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} 0 \\ \frac{1}{2} + \frac{i}{2} \\ \frac{1}{2} - \frac{i}{2} \\ 0 \end{pmatrix}$
4	$\begin{pmatrix} \frac{i}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$	$\frac{1}{4}$	$\begin{pmatrix} \frac{1}{2} + \frac{i}{2} \\ 0 \\ 0 \\ \frac{1}{2} - \frac{i}{2} \end{pmatrix}$