CMSC 442/653
Fall 2007
Instructor: Dr. Lomonaco
Homework 3

- **Reading Assignment:** Review relevant slides on "Overview of Coding Theory" found at http://www.cs.umbc.edu/~lomonaco/f06/653/Slides653.html
- **Reading Assignment:** Read handout Perterson-Pages22-25.pdf found at http://www.cs.umbc.edu/~lomonaco/f06/653/Handouts653.html
- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapter 2, Section 6.

1U) Consider the following degree 4 irreducible polynomial **p(x)** given in Peterson's Table of Irreducible Polynomials over **GF(2)**

DEGREE 4 ... 3 37D ...

a) Write down **p(x)** .
b) Since p(x) is irreducible and of degree 3, it follows that

$$\mathbf{GF(2^4) = GF(2)[x] \bmod p(x)}$$

List all the elements of **GF(2^4)** in the above representation, i.e., in terms of

$$\xi = x \bmod p(x)$$

c) Let $\xi = x \bmod p(x)$. Why is $\left\{\xi^k\right\}$ not a complete list of all the non-zero elements of **GF(2^4)** ?

2U) Consider the following matrix over **GF(2)**

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

a) Prove that the rows of **M** are linearly dependent.
b) Prove that the first three rows **M** form a basis for the row space of **M**.
c) What is the dimension of the row space of **M**? Explain your answer.

3U) Consider the following matrix **S** over **GF(3)**

$$S = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{2} & \mathbf{0} & \mathbf{2} \\ \mathbf{2} & \mathbf{2} & \mathbf{0} & \mathbf{2} & \mathbf{1} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{2} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{1} \end{pmatrix}$$

a) Put the matrix **S** into echelon canonical form. (**Hint.** See section 2.6 of optional text)

b) Use the resulting echelon canonical form to find a basis for the row space of **S** . Explain your answer.

c) What is the dimension of the row space of **S** ? Explain how you found your answer.