# MATH221

quiz #3, 11/13/14

Total 100

Solutions

Show all work legibly. **Name:**________________________

1. (20) Let $M_{2\times2}$ be the vector space of all $2\times2$ matrices. Define $T : M_{2\times2} \to M_{2\times2}$ by $T(A) = A - A^T$.

   (a) (10) True or False? $T$ is a linear transformation.

   **Solution**.

   $$T(\alpha A + \beta B) = \alpha A + \beta B - (\alpha A + \beta B)^T = (\alpha A - \alpha A^T) + (\beta B - \beta B^T) = \alpha T(A) + \beta T(B).$$

   Mark one and explain.

   ▫ True ▫ False

   (b) (10) Describe the kernel of $T$.

   **Solution**. If $0 = T(A) = A - A^T$, then $A = A^T$.

   The kernel of $T$ is:

2. (20) Find a basis for the set of vectors in $\mathbf{R}^2$ on the line $y = 2x$.

   **Solution**. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

   A basis is:

3. Let $p_1(t) = 1 - 3t + 5t^2$, $p_2(t) = -3 + 10t - 7t^2$, $p_3(t) = -4 + 5t - 6t^2$, $p_4(t) = 1 - t^2$. True or False? The set $\{p_1(t), p_2(t), p_3(t), p_4(t)\}$ is linearly independent.

   **Solution**. Consider coordinates of these polynomials

   $$\begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 10 \\ -7 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -6 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

   in the basis $\{1, t, t^2\}$

   $$\begin{bmatrix} 1 & -3 & -4 & 1 \\ -3 & 10 & 5 & 0 \\ 5 & -7 & -6 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -4 & 1 \\ 0 & 1 & -7 & 3 \\ 0 & 8 & 14 & -6 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -4 & 1 \\ 0 & 1 & -7 & 3 \\ 0 & 0 & 70 & -30 \end{bmatrix}.$$

Also note that any 4 vectors in $\mathbf{R}^3$ are linearly dependent.

Mark one and explain.

□ True □ False

4. (20) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} -3 \\ -6 \\ 0 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}, \ \mathbf{v}_4 = \begin{bmatrix} -3 \\ 5 \\ 5 \end{bmatrix}.$$

Find dim span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

**Solution**.

$$\begin{bmatrix} 1 & -3 & -2 & -3 \\ -1 & -6 & 3 & 5 \\ 0 & 0 & 5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & -2 & -3 \\ 0 & -9 & 1 & 2 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

The vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly independent, hence dim span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = 3$.

dim span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} =$

5. (20) If a $4 \times 7$ matrix $A$ has rank 3, find $\dim \text{Null } A$, $\dim \text{Row } A$, and rank $A^T$.

**Solution**. Since rank $A + \dim \text{Null } A = 4$, one has $\dim \text{Null } A = 1$. $\dim \text{Row } A =$ rank $A =$ rank $A^T = 3$.

$\dim \text{Null } A =$

$\dim \text{Row } A =$

rank $A^T$

6. (20) Let $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ be a $2 \times 3$ matrix of rank 1. Suppose that $\mathbf{a}_1 \neq 0$. True or False? There is a vector $\mathbf{c} \in \mathbf{R}^3$ so $\mathbf{a}_1 \mathbf{c}^T = A$.

**Solution**. Since rank $A = 1$ and $\mathbf{a}_1 \neq 0$, $\mathbf{a}_2 = c_2 \mathbf{a}_1$, and $\mathbf{a}_3 = c_3 \mathbf{a}_1$. That is with $\mathbf{c} = \begin{bmatrix} 1 \\ c_2 \\ c_3 \end{bmatrix}$ one has $A = \mathbf{a}_1 \mathbf{c}^T$.

Mark one and explain.

□ False □ True