

**MATH221**

quiz #3, 11/13/14

Total 100

Show all work legibly.

**Name:** \_\_\_\_\_

1. (20) Let  $M_{2 \times 2}$  be the vector space of all  $2 \times 2$  matrices. Define  $T : M_{2 \times 2} \rightarrow M_{2 \times 2}$  by  $T(A) = A - A^T$ .

(a) (10) True or False?  $T$  is a linear transformation.

Mark one and explain.

True       False

(b) (10) Describe the kernel of  $T$ .

The kernel of  $T$  is:

2. (20) Find a basis for the set of vectors in  $\mathbf{R}^2$  on the line  $y = 2x$ .

A basis is:

3. (20) Let  $p_1(t) = 1 - 3t + 5t^2$ ,  $p_2(t) = -3 + 10t - 7t^2$ ,  $p_3(t) = -4 + 5t - 6t^2$ ,  $p_4(t) = 1 - t^2$ . True or False? The set  $\{p_1(t), p_2(t), p_3(t), p_4(t)\}$  is linearly independent.

Mark one and explain.

- True       False

4. (20) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ -6 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} -3 \\ 5 \\ 5 \end{bmatrix}.$$

Find  $\dim \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ .

$$\dim \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} =$$

5. (20) If a  $4 \times 7$  matrix  $A$  has rank 3, find  $\dim \text{Null } A$ ,  $\dim \text{Row } A$ , and  $\text{rank } A^T$ .

$\dim \text{Null } A =$

$\dim \text{Row } A =$

$\text{rank } A^T =$

6. (20) Let  $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3]$  be a  $2 \times 3$  matrix of rank 1. Suppose that  $\mathbf{a}_1 \neq 0$ . True or False? There is a vector  $\mathbf{c} \in \mathbf{R}^3$  so  $\mathbf{a}_1 \mathbf{c}^T = A$ .

Mark one and explain.

False       True