MATH221

quiz #2, 10/14/14

Total 100

Solutions

Show all work legibly. **Name**:______________________

1. (20) Let T be a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^3$ so that $T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, and $T(\mathbf{e}_2) = \begin{bmatrix} -3 \\ 5 \\ 7 \end{bmatrix}$.

   True or False? There is $\mathbf{x} \in \mathbf{R}^2$ so that $T(\mathbf{x}) = \begin{bmatrix} -2 \\ 8 \\ 6 \end{bmatrix}$.

   Solution. To answer the question we have to establish whether the equation $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} -2 \\ 8 \\ 6 \end{bmatrix}$ is consistent.

$$\begin{bmatrix} 1 & -3 & -2 \\ 3 & 5 & 8 \\ -1 & 7 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -2 \\ 0 & 14 & 14 \\ 0 & 4 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

   $x_1 = 1$, and $x_2 = 1$.

   Mark one and explain.

   ▫ False ▫ True

2. (20) True or False? If the equation $A\mathbf{x} = \mathbf{b}$ has two solutions $\mathbf{x}_1 \neq \mathbf{x}_2$, then the columns of A are linearly independent.

   Solution. Let $\mathbf{u} = \mathbf{x}_1 - \mathbf{x}_2 \neq 0$. Since $A\mathbf{x}_1 = \mathbf{b}$, and $A\mathbf{x}_2 = \mathbf{b}$ one has $A\mathbf{u} = 0$, this shows that columns of A are linearly dependent.

   Mark one and explain.

   ▫ True ▫ False

3. (20) Let A and B be two 3×3 matrices, so that the first column of B is all zeros. True or False? The first column of AB is all zeros.

   Solution. Let $B = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$. Note that $AB = [A\mathbf{b}_1, A\mathbf{b}_2, A\mathbf{b}_3]$. If $\mathbf{b}_1 = 0$, then $A\mathbf{b}_1 = 0$.

   Mark one and explain.

   ▫ False ▫ True

4. (20) Find A^{-1}, where

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 4 & -3 & 8 \end{bmatrix}.$$

Solution.

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 4 & -3 & 8 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & -3 & -4 & -4 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & -4 & 3 & 1 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & 3/2 & 1/2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 4 & -2 & -1 \\ 0 & 0 & 1 & -2 & 3/2 & 1/2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 7 & -9/2 & -3/2 \\ 0 & 1 & 0 & 4 & -2 & -1 \\ 0 & 0 & 1 & -2 & 3/2 & 1/2 \end{bmatrix}$$

$A^{-1} =$

5. (20) Let A and B be $n \times n$ matrices. True or False? If AB is invertible, then B is invertible.

Solution. If $B\mathbf{x} = 0$, then $AB\mathbf{x} = 0$. Since AB is invertible $\mathbf{x} = 0$.

Mark one and explain.

□ True □ False

6. (20) Let $T : \mathbf{R}^n \to \mathbf{R}^n$ be a linear transformation so that $T(\mathbf{x}_1) = T(\mathbf{x}_2)$ for a pair of vectors $\mathbf{x}_1 \neq \mathbf{x}_2$. True or False? T is an invertible linear transformation.

Solution. Let A be the standard matrix of T. One has $0 = T(\mathbf{x}_1 - \mathbf{x}_2) = A(\mathbf{x}_1 - \mathbf{x}_2)$. If the inverse transformation T^{-1} exists, then its standard matrix should be A^{-1}, and

$$0 = T^{-1}\left(T(\mathbf{x}_1 - \mathbf{x}_2)\right) = A^{-1}\left(A(\mathbf{x}_1 - \mathbf{x}_2)\right) = \mathbf{x}_1 - \mathbf{x}_2 \neq 0.$$

This contradiction shows that T^{-1} does not exist.

Mark one and explain.

□ True □ False