MATH221

quiz #1, 09/23/14

Total 100

Solutions

Show all work legibly. **Name**:______________________

1. (20) Solve the system

$$\begin{array}{rrrcl} 2x_1 & & -4x_3 & = & 0 \\ & x_2 & +3x_3 & = & 2 \\ x_1 & +5x_2 & +8x_3 & = & 0 \end{array}$$

Solution.

$$\begin{bmatrix} 2 & 0 & -4 & 0 \\ 0 & 1 & 3 & 2 \\ 1 & 5 & 8 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 1 & 5 & 8 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 5 & 10 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & -5 & -10 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 = 4,\ x_2 = -4,\ x_3 = 2$

2. (20) Determine values of h for which the system

$$2x_1 - 6x_2 = h,\ -4x_1 + 12x_2 = 2$$

has no solutions.

Solution.

$$\begin{bmatrix} 2 & -6 & h \\ -4 & 12 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -6 & h \\ 0 & 0 & 2+2h \end{bmatrix}$$

$h \neq -1$

3. (20) Let

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix},$$

and let W be the set of all linear combinations of the columns of A. True or False? The last column of A is in W.

Solution.
$$\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} = 0 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix} + 1 \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}.$$

Mark one and explain.

□ True □ False

4. (20) True or False? If A is 5×3 matrix, $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix}$, and $A\mathbf{y} = \mathbf{b}$, then the equation $A\mathbf{x} = -2\mathbf{b} = \begin{bmatrix} 0 \\ -10 \\ -12 \end{bmatrix}$ is consistent.

Solution. If If A is 5×3 matrix, then $A\mathbf{y}$ is a vector with 5 entries, and $A\mathbf{y}$ may not be equal to a vector with 3 entries (full credit will be given for this statement). If $\mathbf{b}$ is a vector with 5 entries, then $A(-2\mathbf{y}) = -2A\mathbf{y} = -2\mathbf{b}$.

Mark one and explain.

□ True, $\mathbf{x} =$ □ False

5. (20) True or False? The vectors
$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -5 \\ -3 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
are linearly dependent.

Solution. $0 \cdot \mathbf{v}_1 + 0 \cdot \mathbf{v}_2 + 1 \cdot \mathbf{v}_3 = 0$

Mark one and explain.

□ True □ False

6. (20) True or False? If vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} \subset \mathbf{R}^5$ are linearly dependent, then the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{0}\}$ are also linearly dependent.

Solution. If $0 \cdot \mathbf{v}_1 + 0 \cdot \mathbf{v}_2 + 0 \cdot \mathbf{v}_3 + 0 \cdot \mathbf{v}_4 + 1 \cdot \mathbf{0} = 0$.

Mark one and explain.

□ True □ False