MATH221

Final Examination

December 16, 2014

Total 140

1. (20) Let $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$. Find a basis for the null space of A.

The basis for the null space of A is:

2. (20) Evaluate the determinant det A of the matrix $\begin{bmatrix} 2 & 2 & -3 & 1 \\ 0 & 1 & 2 & -1 \\ 3 & -1 & 4 & 1 \\ 2 & 3 & 0 & 0 \end{bmatrix}$.

det $A =$

3. (20) Consider $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 3 \\ -1 \end{bmatrix} \right\}$. (Note that $\mathcal{B}$ is an orthogonal set of vectors.)

- (10) Find the vector $\mathbf{x}$ in the subspace $W = span(\mathcal{B})$ whose coordinate vector relative to the basis $\mathcal{B}$ is the vector $(-2, 5, 3)^T$.

$\mathbf{x} =$

- (10) The vector $\mathbf{v} = (24, 17, 29, -3)^T$ lies in the subspace W. Find its coordinates relative to the basis $\mathcal{B}$.

$[\mathbf{v}]_{\mathcal{B}} =$

4. (20) Consider the linear system of equations

$$\begin{array}{rcrcrcr} x & - & y & + & z & = & a \\ 2x & + & y & - & 3z & = & 8 \\ x & - & 2y & + & 3z & = & -5 \end{array}$$

It is given that $x = 1$; find the value of a.

$a =$

5. (20) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

- (10) Find a matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix.

$P =$

- (10) Compute D.

$D =$

6. (20) $A_{n\times n}$ matrix satisfies $\det\left(A^3\right) = -1$. Find $\det A$.

$\det A =$

7. (20) True or False. If $A_{2\times 2}$ matrix satisfying $A^2 = 0$, then

(a) **(10 pts.)** $A = 0$.

Mark one and explain.

□ True □ False

(b) **(10 pts.)** $\det A = 0$.

Mark one and explain.

□ True □ False