MATH221

quiz #3, 11/12/13

Total 100

Solutions

Show all work legibly. **Name**:______________________

1. (20) Let $p_1(x) = 2$, $p_2(x) = 2 + 3x$, and $p_3(x) = 2 + 3x + 3x^2$.

   - (10) True or False? The vector set $\{p_1, p_2, p_3\}$ is linearly independent.

     Solution. Consider a basis $\mathcal{B} = \{1, x, x^2\}$ in P_2. Note that

     $$[p_1]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \ [p_2]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \ [p_3]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}.$$

     Since the vectors $[p_1]_{\mathcal{B}}$, $[p_2]_{\mathcal{B}}$, $[p_3]_{\mathcal{B}}$ are linearly independent the vector set $\{p_1, p_2, p_3\}$ is linearly independent.

     Mark one and explain.

     ▫ True ▫ False

   - (10) True or False? The vector set $\{p_1, p_2, p_3\}$ spans P_2.

     Solution. Three linearly independent vectors $\{p_1, p_2, p_3\}$ (see the first part of the problem) span the space P_2.

     Mark one and explain.

     ▫ True ▫ False

2. (20) Let A be an 2×3 matrix so that the equation $A^T\mathbf{x} = 0$ has only the trivial solution. True or False? $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b} \in \mathbf{R}^2$.

   Solution. If $A^T\mathbf{x} = 0$ always implies $\mathbf{x} = 0$, then 2 rows of A are linearly independent, and rank $A = 2$. This means that 2 columns of A are linearly independent, hence these columns span $\mathbf{R}^2$.

   Mark one and explain.

   ▫ True ▫ False

3. (20) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 6 & 9 \end{bmatrix}$. Compute det A.

Solution.

$$\det A = 3 \times \det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} = 0.$$

det $A =$

4. (20) Let A be a 2×2 matrix such that det $A^2 = 0$. True or False? A is invertible.

Solution. Since $0 = \det\,(A \times A) = \det A \times \det A$, one has $\det A = 0$, and A is not invertible.

Mark one and explain.

□ True □ False

5. (20) Let A be a 2×2 matrix such that $A^T A = I$. True or False? A is invertible.

Solution. Since $1 = \det I = \det\,(A^T A) = \det A^T \times \det A$, one has $\det A \neq 0$, and A is invertible.

6. (20) Let $M_{2\times 2}$ be the vector space of all 2×2 matrices. Define $T : M \to M$ by $T(A) = A + A^T$. True or False? T is a linear transformation.

Solution. The transformation $T_1(A) = A$ is linear. The transformation $T_2(A) = A^T$ satisfies:

(a) $T_2(cA) = (cA)^T = cA^T$,

(b) $T_2(A + B) = (A + B)^T = A^T + B^T$.

Hence the transformation T_2 is linear. The sum of two linear transformations $T = T_1 + T_2$ is a linear transformation.

Mark one and explain.

□ True □ False