# MATH221

quiz #2, 10/17/13

Total 100

Solutions

Show all work legibly. **Name**:________________________

1. (20) Let $T$ be a linear transformation that reflects a vector in $\mathbf{R}^2$ with respect to the $45^o$ line $y = x$.

   (a) (10) Find $A$ the standard matrix for the linear transformation $T$.

   **Solution**. $A = [T(\mathbf{e}_1)T(\mathbf{e}_2)]$. Note that $T(\mathbf{e}_1) = \mathbf{e}_2$, and $T(\mathbf{e}_2) = \mathbf{e}_1$. Hence $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

   $A =$

   (b) (10) True or False? $T$ is one–to–one.

   **Solution**. If $T\mathbf{x} = 0$, then $x_2 = x_1 = 0$. Hence $T$ is one–to–one.

   Mark one and explain.

   ▫ True ▫ False

2. (20) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

   (a) (10) Find $A^{-1}$ if exists.

   $A^{-1} = A$.

   (b) (10) If $B$ is a $2 \times 3$ matrix so that $AB = C = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$. Find $B$.

   **Solution**. $AB = C$, hence $B = A^{-1}C = \begin{bmatrix} 3 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix}$.

   $B =$

3. (20) Let $A$ be a $2 \times 2$ matrix so that the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b}$. True or False? For some $\mathbf{b}$ the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution.

   **Solution**. If $A\mathbf{x}_i = \mathbf{e}_i$, $i = 1, 2$, then $A[\mathbf{x}_1, \mathbf{x}_2] = I$, and $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2] = A^{-1}$. Suppose there is $\mathbf{b}$ such that $A\mathbf{y}_1 = \mathbf{b}$ and $A\mathbf{y}_2 = \mathbf{b}$ for $\mathbf{y}_1 \neq \mathbf{y}_2$. Note that $\mathbf{y} = \mathbf{y}_1 - \mathbf{y}_2 \neq \mathbf{0}$, $A\mathbf{y} = \mathbf{0}$, and

   $$0 \neq \mathbf{y} = A^{-1}A\mathbf{y} = A^{-1}\mathbf{0} = \mathbf{0}.$$

   This contradiction shows that the assumption about existence of $\mathbf{b}$ is wrong.

Mark one and explain.

□ True □ False

4. (20) Suppose a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ has the property $T(\mathbf{u}) = T(\mathbf{v})$ for any pair of vectors $\mathbf{u},\ \mathbf{v} \in \mathbf{R}^3$. Find $A$, the standard matrix of $T$.

   **Solution**. Since for any vector $\mathbf{x}$ one has $T(\mathbf{x}) = T(\mathbf{0}) = 0$ the matrix $A$ is the zero matrix.

   $A =$

5. (20) Let $V$ be a vector space of all real valued functions of one real variable, i.e.

$$V = \{f\ :\ f\ :\ \mathbf{R} \to \mathbf{R}\}.$$

   Consider the transformation of $V$ into $\mathbf{R}$ defined by $T(f) = f(1)$. True or False? $T$ is a linear transformation.

   **Solution**. For two functions $f$ and $g$, and two scalars $a$ and $b$ one has

$$T(af + bg) = af(1) + bg(1) = aT(f) + bT(g).$$

   Mark one and explain.

   □ True □ False

6. (20) Let $V$ and $W$ be vector spaces, and let $T\ :\ V \to W$ be a linear transformation. Given a subspace $U$ of $V$, let $T(U)$ denote the set of all images of the form $T(\mathbf{x})$ where $\mathbf{x} \in U$, i.e. $T(U) = \{\mathbf{y}\ :\ \mathbf{y} = T(\mathbf{x}) \text{ for some } \mathbf{x} \in V\}$. True or False? $T(U)$ is a subspace of $W$.

   **Solution**. If $\mathbf{y}_1,\ \mathbf{y}_2 \in T(U)$, then there are $\mathbf{x}_1,\ \mathbf{x}_2 \in V$ such that $T(\mathbf{x}_1) = \mathbf{y}_1$, and $T(\mathbf{x}_2) = \mathbf{y}_2$. Hence

$$c_1\mathbf{y}_1 + c_2\mathbf{y}_2 = T(c_1\mathbf{x}_1 + c_2\mathbf{x}_2) \in T(U).$$

   Mark one and explain.

   □ True □ False