Name:

**MATH221**
quiz #1, 09/28/11
Total 100
**Solutions**

Show all work legibly.

1. (20) Solve the system:

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & + & x_3 & = & 0 \\ & & 2x_2 & - & 8x_3 & = & 8 \\ -4x_1 & + & 5x_2 & + & 9x_3 & = & 1 \end{array}$$

**Solution**.

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -3 & 13 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & -3 & 13 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & 13 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & -2 & 0 & -13 \\ 0 & 1 & 0 & 56 \\ 0 & 0 & 1 & 13 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 99 \\ 0 & 1 & 0 & 52 \\ 0 & 0 & 1 & 13 \end{bmatrix}$$

$x_1 = 99$ $x_2 = 56$ $x_3 = 13$

2. (20) Write a system of equations that is equivalent to the vector equation

$$x_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}.$$

**Solution**. The system of equations is:

$$6x_1 + 2x_3 = 1,\ -x_1 + 4x_2 + x_3 = -7,\ 5x_1 = -5.$$

3. (20) Determine the values of $k$ and $h$ for which the system

$$2x_1 - 6x_2 = k,\ -4x_1 + 12x_2 = h$$

is consistent.

**Solution**.

$$\begin{bmatrix} 2 & -6 & k \\ -4 & 12 & h \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -6 & k \\ 0 & 0 & h+2k \end{bmatrix}$$

The system is consistent if $h + 2k = 0$

4. (20) Let

$$A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3] = \begin{bmatrix} 2 & 0 & -4 \\ -1 & 8 & 2 \\ 1 & -2 & -2 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 0 \end{bmatrix}.$$

- (15) True or False? $\mathbf{b}$ is in the set of all linear combinations of the columns of $A$.

  **Solution**.

$$\begin{bmatrix} 2 & 0 & -4 & 10 \\ -1 & 8 & 2 & 3 \\ 1 & -2 & -2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & -2 & 0 \\ 2 & 0 & -4 & 10 \\ -1 & 8 & 2 & 3 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & -2 & -2 & 0 \\ 0 & 4 & 0 & 10 \\ 0 & 6 & 0 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 5/2 \\ 0 & 6 & 0 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 0 & -12 \end{bmatrix}.$$

  Mark one and explain.

  ▫ True ▫ False

- True or False? span $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\} = \text{span}\ \{\mathbf{a}_1, \mathbf{a}_2\}$

  **Solution**. Note that $-2\mathbf{a}_1 = \mathbf{a}_3$.

  Mark one and explain.

  ▫ True ▫ False

5. (20) Let $A$ be a $2 \times 2$ matrix, such that

$$A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \text{ and } A \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}.$$

Compute $A \begin{bmatrix} 7 \\ 10 \end{bmatrix}$.

**Solution**.

$$\begin{bmatrix} 7 \\ 10 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix}, \text{ hence } A \begin{bmatrix} 7 \\ 10 \end{bmatrix} = \begin{bmatrix} 13 \\ 16 \end{bmatrix}.$$

6. (20) True or False? If $A$ is a matrix such that $A\mathbf{x} = 0$ has a unique solution, then for each $\mathbf{b}$ the system $A\mathbf{x} = \mathbf{b}$ has a unique solution.

**Solution**.
Let $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The equation $A\mathbf{x} = 0$ has a unique solution, but $A\mathbf{x} = \mathbf{b}$ has no solution.

Mark one and explain.

▫ True ▫ False