Name:________________________________

SECTION	Points
1.	/10
2.	/14
3.	/14
4.	/14
5.	/16
6.	/16
7.	/16
TOTAL:	/100

Instructions:

- This is a closed-book, closed-notes exam.
- You may use a calculator for this exam. Your calculator must not have an internet connection.
- Answer all questions as fully as you can. Partial credit will be given for intelligent and intelligible answers.
- You have 75 minutes for the exam.

1 True/False: Functions and Sets (10 pts)

Circle TRUE or FALSE for each of the following statements. Pay close attention to the domain and codomain (a.k.a. range) of the function in each question. The set of natural numbers, integers and real numbers are denoted by $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$ respectively.

a. For $f : \mathbb{N} \to \mathbb{N}$, $f(n) = n + 13$ is one-to-one.

TRUE FALSE

b. For $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = n + 13$ is one-to-one.

TRUE FALSE

c. For $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = 5n - 2$ is onto.

TRUE FALSE

d. For $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 5x - 2$ is onto.

TRUE FALSE

e. For $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = 5n + 2$ is a bijection.

TRUE FALSE

f. For $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 5x + 2$ is a bijection.

TRUE FALSE

g. For all sets A and B, $(A \subseteq B) \Rightarrow (\overline{A} \subseteq \overline{B})$.

TRUE FALSE

h. For all sets A, B and C, $(A \subseteq B) \Rightarrow (A \cup C \subseteq B \cup C)$.

TRUE FALSE

i. For all sets A and B, $A \cup B = (A - B) \cup (B - A)$

TRUE FALSE

j. For all sets A and B, $A \cup (A \cap B) = A$

TRUE FALSE

2 Logic: Truth Tables (14 pts)

Use a truth table to show that the following proposition is a tautology.

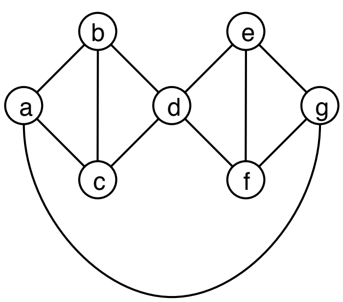
$$\neg(p \Rightarrow \neg q) \vee \neg(p \wedge q)$$

You must show the intermediate steps in the truth table.

p	q	
F	F	
F	T	
T	F	
T	T	

3 Proofs: Indirect Proof (14 pts)

Recall that an undirected graph is 3-colorable if we can color each vertex of the graph either red, blue or green in such way that no two adjacent vertices have the same color. Provide a *well-written* indirect proof that the following graph is not 3-colorable. Your proof must follow the indirect proof format.



6 Equivalence Proof: Sets (16 pts)

Provide a *well-written* equivalence proof that

$$(A - B) - C = A \cap \overline{(B \cup C)}.$$

You must prove the equality of the two sets by showing that every element of the set on the left hand side of the equality is also an element of the set on the right hand side, and vice versa. (I.e., do not prove this using algebraic identities.)

7 Graphs: Bipartite Graphs (16 pts)

We say that an undirected graph G with vertex set V and edge set E is *bipartite* if it is possible to divide V into two subsets L and R such that every vertex is in exactly one of L and R (i.e., such that $L \cup R = V$ and $L \cap R = \emptyset$) and every edge in E is incident upon a vertex in L and a vertex in R.

Find a division of the vertices of the graph below into L and R which shows that the graph is bipartite. First, circle the vertices in the L. Second, redraw the graph by placing the vertices in L on the left side of the diagram, the vertices of R on the right side of the diagram and then draw in all the edges in E. Remember to label each vertex in your diagram.