Due: Tuesday March 24, 2009

Mathematical Induction.   In the following question you are asked to use proof by induction. Your proof must not simply be a sequence of equations, even if the statement you are proving is arithmetic in nature. Clearly indicate using complete English sentences: 1) what you are allowed to assume from the induction hypothesis, 2) what you need to show to establish the induction step, and 3) which steps of the proof uses the induction hypothesis.

1. Prove by induction that for all integers $n \geq 1$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2} \right]^2 .$$

2. Prove by induction that for all integers $n \geq 1$

$$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \frac{1}{9 \cdot 13} + \cdots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1} .$$

3. Prove by induction that for all integers $n \geq 2$

$$\left(1 - \frac{1}{2^2} \right) \cdot \left(1 - \frac{1}{3^2} \right) \cdot \left(1 - \frac{1}{4^2} \right) \cdots \cdot \left(1 - \frac{1}{n^2} \right) = \frac{n+1}{2n} .$$