CMSC 471
Fall 2012

Class #11

Thursday, October 4
Propositional Logic

Kevin Winner, winnerk1@umbc.edu

This class

- HW2 submission details
- Propositional logic
- HW3

Propositional Logic

Chapter 7.4-7.8

Some material adopted from notes
by Andreas Geyer-Schulz
and Chuck Dyer

Propositional Logic

- **Logical constants**: true, false
- **Propositional symbols**: P, Q, S, ... (**atomic sentences**)
- Wrapping **parentheses**: (…)
- Sentences are combined by **connectives**:

$\wedge$ ...and	[conjunction]
$\vee$ ...or	[disjunction]
$\Rightarrow$...implies	[implication / conditional]
$\Leftrightarrow$..is equivalent	[biconditional]
$\neg$ ...not	[negation]

- **Literal**: atomic sentence or negated atomic sentence

Truth Tables II

The five logical connectives:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
False	*False*	*True*	*False*	*False*	*True*	*True*
False	*True*	*True*	*False*	*True*	*True*	*False*
True	*False*	*False*	*False*	*True*	*False*	*False*
True	*True*	*False*	*True*	*True*	*True*	*True*

A complex sentence:

P	H	$P \vee H$	$(P \vee H) \wedge \neg H$	$((P \vee H) \wedge \neg H) \Rightarrow P$
False	*False*	*False*	*False*	*True*
False	*True*	*True*	*False*	*True*
True	*False*	*True*	*True*	*True*
True	*True*	*True*	*False*	*True*

Propositional Logic (PL)

- A simple language useful for showing key ideas and definitions
- User defines a set of propositional symbols, like P and Q.
- User defines the **semantics** of each propositional symbol:
  – Ho means "It is hot"
  – Hu means "It is humid"
  – R means "It is raining"
- A sentence (well formed formula) is defined as follows:
  – A symbol is a sentence
  – If S is a sentence, then $\neg S$ is a sentence
  – If S is a sentence, then (S) is a sentence
  – If S and T are sentences, then $S \vee T$, $S \wedge T$, $S \rightarrow T$, and $S \leftrightarrow T$ are sentences
  – A sentence results from a finite number of applications of the above rules

Examples of PL Sentences

- $(P \wedge Q) \rightarrow R$

"If it is hot and humid, then it is raining"

- $Q \rightarrow P$

"If it is humid, then it is hot"

- Q

"It is humid."

- A better way:

Ho = "It is hot"

Hu = "It is humid"

R = "It is raining"

Some Terms

- The meaning or **semantics** of a sentence determines its **interpretation**.
- Given the truth values of all symbols in a sentence, it can be "evaluated" to determine its **truth value** (True or False).
- A **model** for a KB is a "possible world" (assignment of truth values to propositional symbols) in which each sentence in the KB is True.

More Terms

- A **valid sentence** or **tautology** is a sentence that is True under all interpretations, no matter what the world is actually like or what the semantics are. Example: "It's raining or it's not raining."
- An **inconsistent sentence** or **contradiction** is a sentence that is False under all interpretations. The world is never like what it describes, as in "It's raining and it's not raining."

Two Important Properties for Inference

Soundness: If KB |- Q then KB |= Q

- If Q is derived from a set of sentences KB using a given set of rules of inference, then Q is entailed by KB.
- Hence, inference produces only real entailments, or any sentence that follows deductively from the premises is valid.

Completeness: If KB |= Q then KB |- Q

- If Q is entailed by a set of sentences KB, then Q can be derived from KB using the rules of inference.
- Hence, inference produces all entailments, or all valid sentences can be proved from the premises.

Sound Rules of Inference

- Here are some examples of sound rules of inference
  - *A rule is sound if its conclusion is true whenever the premise is true*
- Each can be shown to be sound using a truth table

RULE	PREMISE	CONCLUSION
Modus Ponens	A, A → B	B
And Introduction	A, B	A ∧ B
And Elimination	A ∧ B	A
Double Negation	¬ ¬ A	A
Unit Resolution	A ∨ B, ¬ B	A
Resolution	**A ∨ B, ¬ B ∨ C**	**A ∨ C**

Soundness of Modus Ponens

A	B	A → B	OK? $(A \wedge (A \rightarrow B)) \rightarrow B$
True	True	True	√
True	False	False	√
False	True	True	√
False	False	True	√

Soundness of the Resolution Inference Rule

$(\alpha \vee \beta) \wedge (\sim\beta \vee \gamma) \rightarrow (\alpha \vee \gamma)$

α	β	γ	$\alpha \vee \beta$	$\neg\beta \vee \gamma$	$\alpha \vee \gamma$
False	*False*	*False*	*False*	*True*	*False*
False	*False*	*True*	*False*	*True*	*True*
False	*True*	*False*	*True*	*False*	*False*
False	*True*	*True*	*True*	*True*	*True*
True	*False*	*False*	*True*	*True*	*True*
True	*False*	*True*	*True*	*True*	*True*
True	*True*	*False*	*True*	*False*	*True*
True	*True*	*True*	*True*	*True*	*True*

Proving Things

- A **proof** is a sequence of sentences, where each sentence is either a premise or a sentence derived from earlier sentences in the proof by one of the rules of inference.
- The last sentence is the **theorem** (also called goal or query) that we want to prove.
- Example for the "weather problem" given above.

1. Hu	Premise	"It is humid"
2. $Hu \rightarrow Ho$	Premise	"If it is humid, it is hot"
3. Ho	Modus Ponens(1,2)	"It is hot"
4. $(Ho \wedge Hu) \rightarrow R$	Premise	"If it's hot & humid, it's raining"
5. $Ho \wedge Hu$	And Introduction(1,3)	"It is hot and humid"
6. R	Modus Ponens(4,5)	"It is raining"

Proof by Resolution

- If our knowledge base consists only of **clauses** (disjunctions of literals), then we can form a **complete** resolution algorithm using any complete search algorithm combined with the **resolution rule**:

  $$\mathbf{A} \lor \mathbf{B},\ \neg\mathbf{B} \lor \mathbf{C} \rightarrow \mathbf{A} \lor \mathbf{C}$$

- That's great, but what if our knowledge base has more than just clauses?
  - A clause-only KB is said to be in **conjunctive normal form (CNF)**
  - Any propositional logic KB can be converted to CNF

Converting to CNF

1. Eliminate $\leftrightarrow$ by replacing $A \leftrightarrow B$ with $(A \rightarrow B) \wedge (B \rightarrow A)$
2. Eliminate $\rightarrow$ by replacing $A \rightarrow B$ with $\neg A \vee B$
3. Move $\neg$ inwards by applying:
   - $\neg(\neg A) = A$
   - $\neg(A \wedge B) = (\neg A \vee \neg B)$
   - $\neg(A \vee B) = (\neg A \wedge \neg B)$
4. Distribute $\vee$ over $\wedge$ whenever possible with:
   - $(A \vee (B \wedge C)) = ((A \vee B) \wedge (A \vee C))$

Proof by Resolution

- Given our CNF formatted KB, we can build a sound and complete resolution algorithm
  - Add our query (in the form of a clause) to the KB
  - For each pair of clauses in the KB, apply the resolution rule to both of them
    - Add any derived sentences to the KB
    - If we ever derive the empty clause (which is always false), then the KB is inconsistent, return false
    - If we can no longer derive new sentences via resolution, then the query is consistent, return true

Propositional Logic is a Weak Language

- Can't directly talk about properties of individuals or relations between individuals (e.g., "Bill is tall")
- Generalizations, patterns, regularities can't easily be represented (e.g., "all triangles have 3 sides")
- First-Order Logic (abbreviated FOL or FOPC) is expressive enough to concisely represent this kind of information
- FOL adds relations, variables, and quantifiers, e.g.,
  - *"Every elephant is gray":* $\forall$ x (elephant(x) $\rightarrow$ gray(x))
  - *"There is a white alligator":* $\exists$ x (alligator(X) ^ white(X))

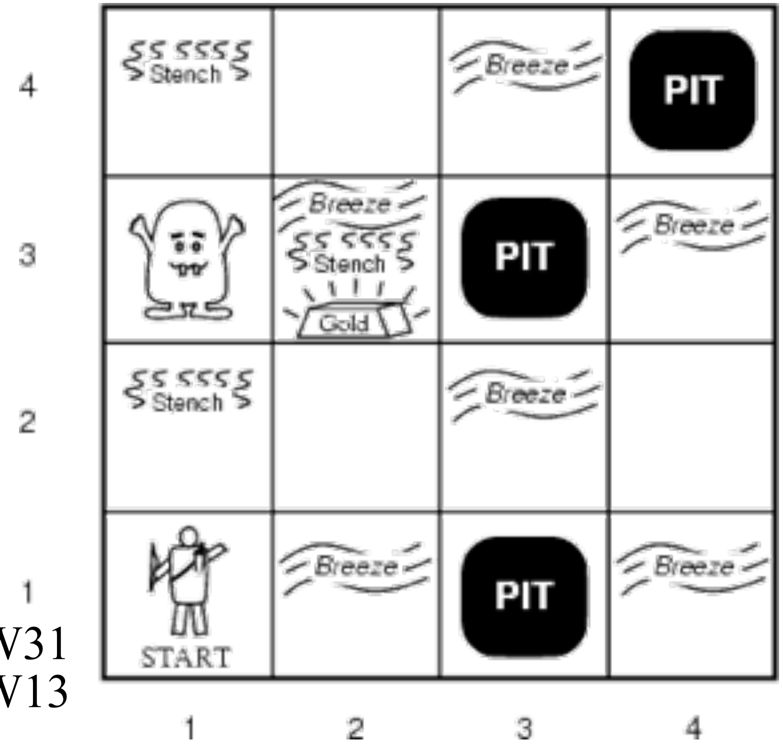
The "Hunt the Wumpus" Agent

- Some atomic propositions:

S12 = There is a stench in cell (1,2)
B34 = There is a breeze in cell (3,4)
W13 = The Wumpus is in cell (1,3)
V11 = We have visited cell (1,1)
OK11 = Cell (1,1) is safe.
etc

- Some rules:

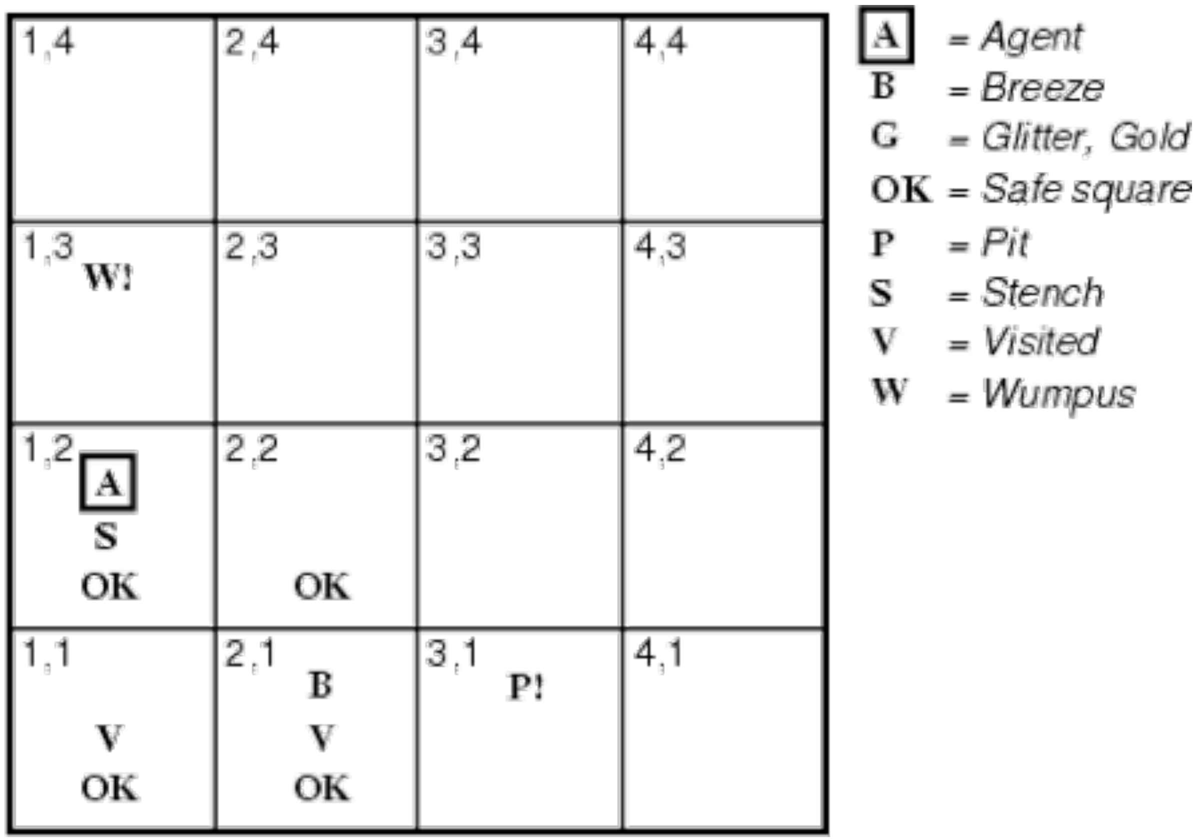
(R1) $\neg S11 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W21$
(R2) $\neg S21 \rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$
(R3) $\neg S12 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$
(R4) $S12 \rightarrow W13 \vee W12 \vee W22 \vee W11$
etc.



- Note that the lack of variables requires us to give similar rules for each cell

After the Third Move

- We can prove that the Wumpus is in (1,3) using the four rules given.
- See R&N section 7.5



Proving W13

- Apply MP with $\neg$S11 and R1:

  $\neg W11 \wedge \neg W12 \wedge \neg W21$
- Apply And-Elimination to this, yielding three sentences:

  $\neg W11, \neg W12, \neg W21$
- Apply MP to ~S21 and R2, then apply And-Elimination:

  $\neg W22, \neg W21, \neg W31$
- Apply MP to S12 and R4 to obtain:

  $W13 \vee W12 \vee W22 \vee W11$
- Apply Unit Resolution on $(W13 \vee W12 \vee W22 \vee W11)$ and $\neg W11$:

  $W13 \vee W12 \vee W22$
- Apply Unit Resolution with $(W13 \vee W12 \vee W22)$ and $\neg W22$:

  $W13 \vee W12$
- Apply UR with $(W13 \vee W12)$ and $\neg W12$:

  W13
- QED

Summary

- The process of deriving new sentences from old one is called **inference**.
  - **Sound** inference processes derives true conclusions given true premises
  - **Complete** inference processes derive all true conclusions from a set of premises
- A **valid sentence** is true in all worlds under all interpretations
- If an implication sentence can be shown to be valid, then—given its premise—its consequent can be derived
- Different logics make different **commitments** about what the world is made of and what kind of beliefs we can have regarding the facts
  - Logics are useful for the commitments they do not make because lack of commitment gives the knowledge base engineer more freedom
- **Propositional logic** commits only to the existence of facts that may or may not be the case in the world being represented
  - It has a simple syntax and simple semantics. It suffices to illustrate the process of inference
  - Propositional logic quickly becomes impractical, even for very small worlds