Red-Black Trees

Bottom-Up Deletion

Recall "ordinary" BST Delete

1. If node to be deleted is a leaf, just delete it.
2. If node to be deleted has just one child, replace it with that child (splice)
3. If node to be deleted has two children, replace the **value** in the node by its in-order predecessor/successor's value then delete the in-order predecessor/successor (a recursive step)

Bottom-Up Deletion

1. Do ordinary BST deletion. Eventually a "case 1" or "case 2" deletion will be done (leaf or just one child).
   -- If deleted node, U, is a leaf, think of deletion as replacing U with the NULL pointer, V.
   -- If U had one child, V, think of deletion as replacing U with V.
2. What can go wrong??

Which RB Property may be violated after deletion?

1. If U is Red?

   Not a problem – no RB properties violated

2. If U is Black?

   If U is not the root, deleting it will change the black-height along some path

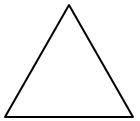
Fixing the problem

- Think of V as having an “extra” unit of blackness. This extra blackness must be absorbed into the tree (by a red node), or propagated up to the root and out of the tree.
- There are four cases – our examples and “rules” assume that V is a left child. There are symmetric cases for V as a right child.

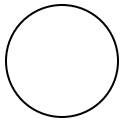
Terminology

- The node just deleted was U
- The node that replaces it is V, which has an extra unit of blackness
- The parent of V is P
- The sibling of V is S

Black Node



Red or Black and don't care

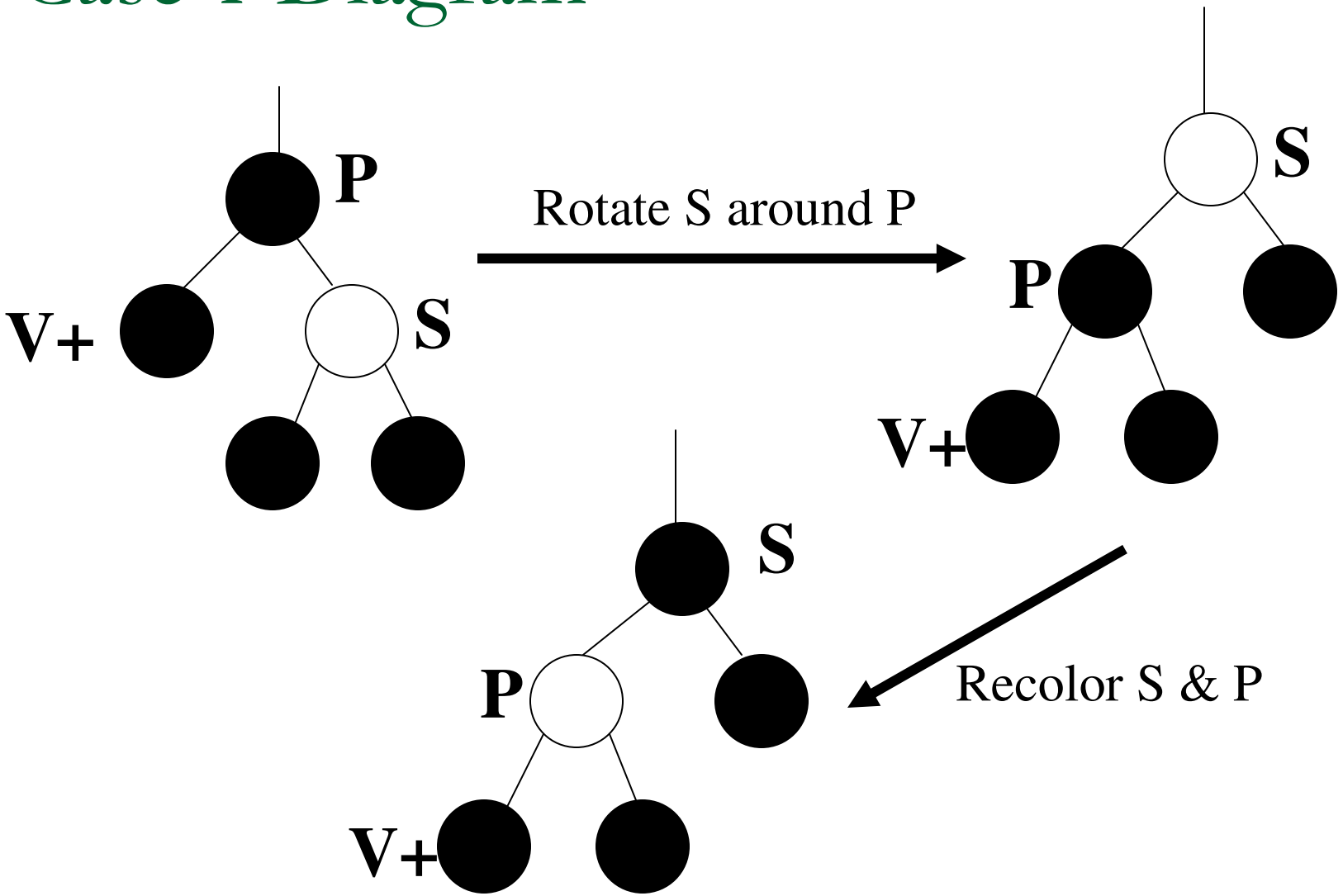


Red Node

Bottom-Up Deletion
Case 1

- V's sibling, S, is Red
  - Rotate S around P and recolor S & P
- NOT a terminal case – One of the other cases will now apply
- All other cases apply when S is Black

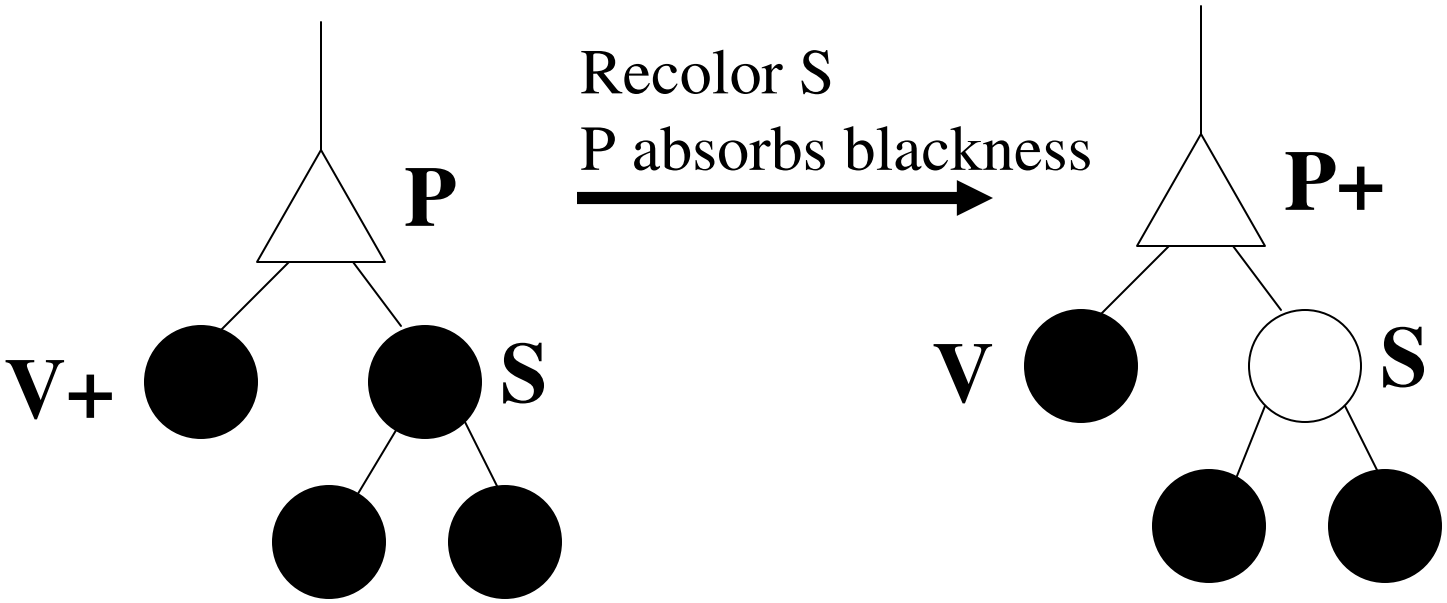
Case 1 Diagram



Bottom-Up Deletion Case 2

- V's sibling, S, is Black and has two Black children.
  - Recolor S to be Red
  - P absorbs V's extra blackness
    - If P is Red, we're done (it absorbed the blackness)
    - If P is Black, it now has extra blackness and problem has been propagated up the tree

Case 2 diagram



Either extra Black absorbed by P

or

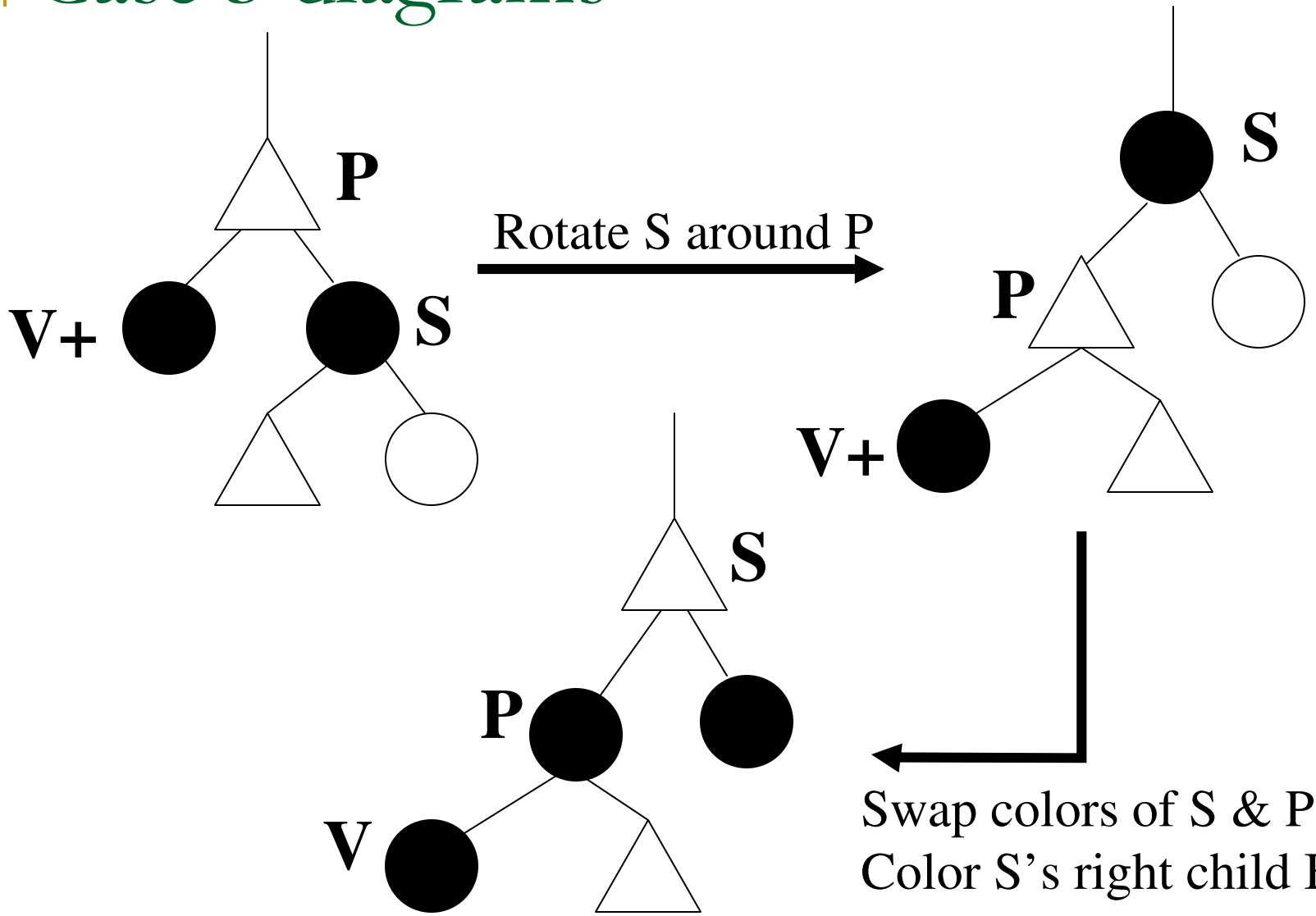
P now has extra blackness

Bottom-Up Deletion
Case 3

- S is Black
- S's right child is RED (Left child either color)
  - Rotate S around P
  - Swap colors of S and P,
    and color S's right child Black

- This is the terminal case – we're done

Case 3 diagrams

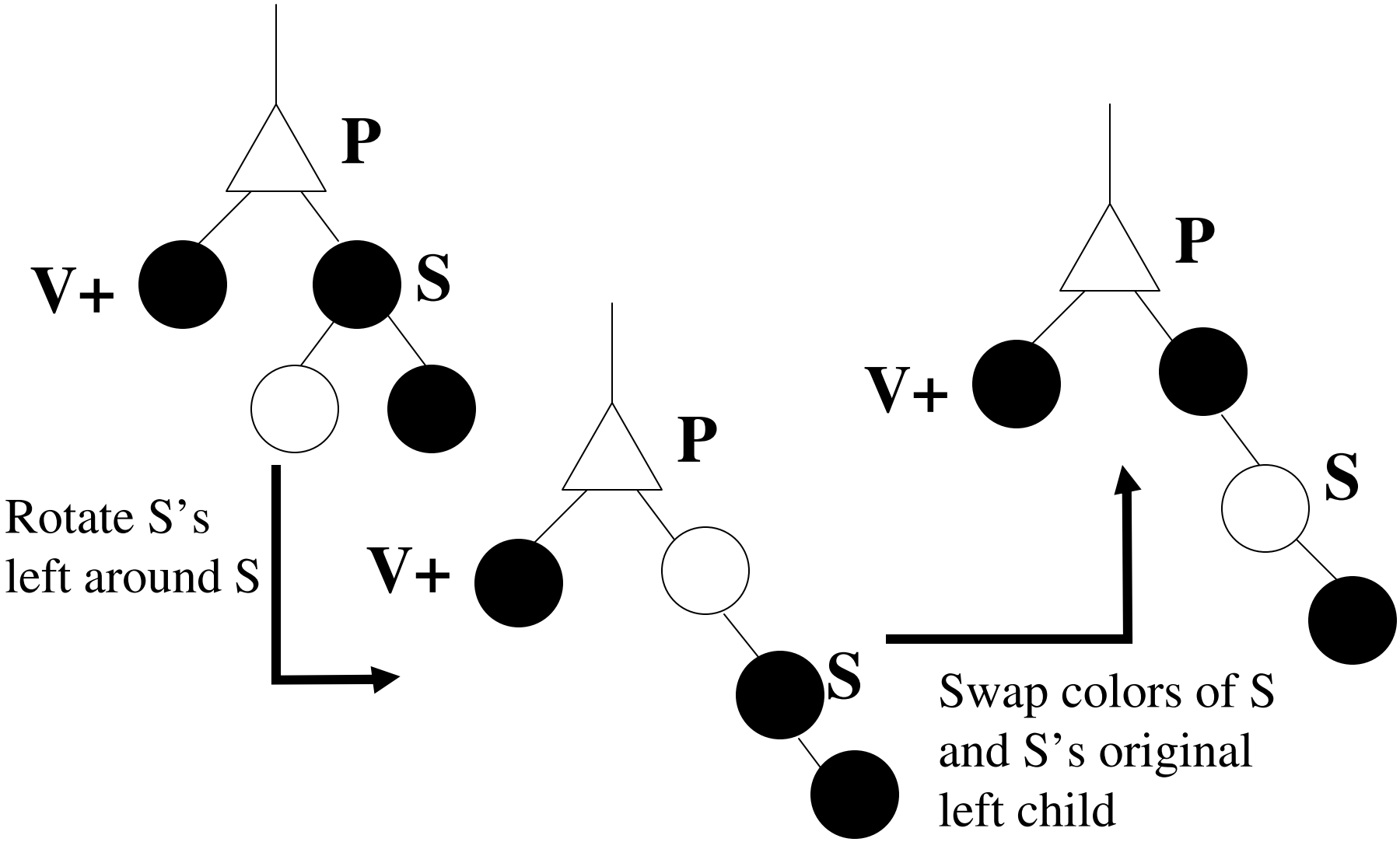


Bottom-Up Deletion

Case 4

- S is Black, S's right child is Black and S's left child is Red
  - Rotate S's left child around S
  - Swap color of S and S's left child
  - Now in case 3

Case 4 Diagrams

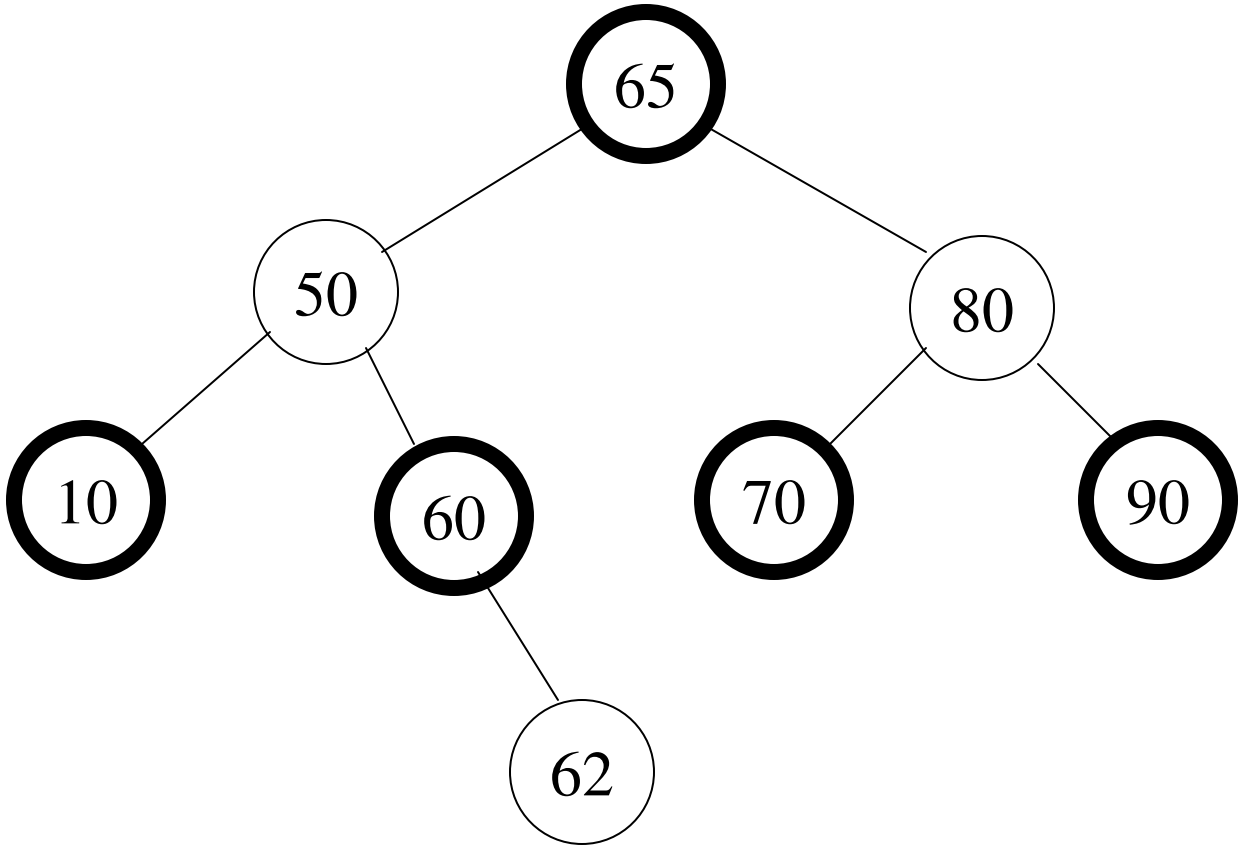


Top-Down Deletion

An alternative to the recursive “bottom-up” deletion is “top-down” deletion.

This method is iterative. It moves down the tree only, “fixing” things as it goes.

What is the goal of top-down deletion?



Perform the following deletions, in the order specified
Delete 90, Delete 80, Delete 70