CMSC 341

Binary Search Trees

Binary Search Tree

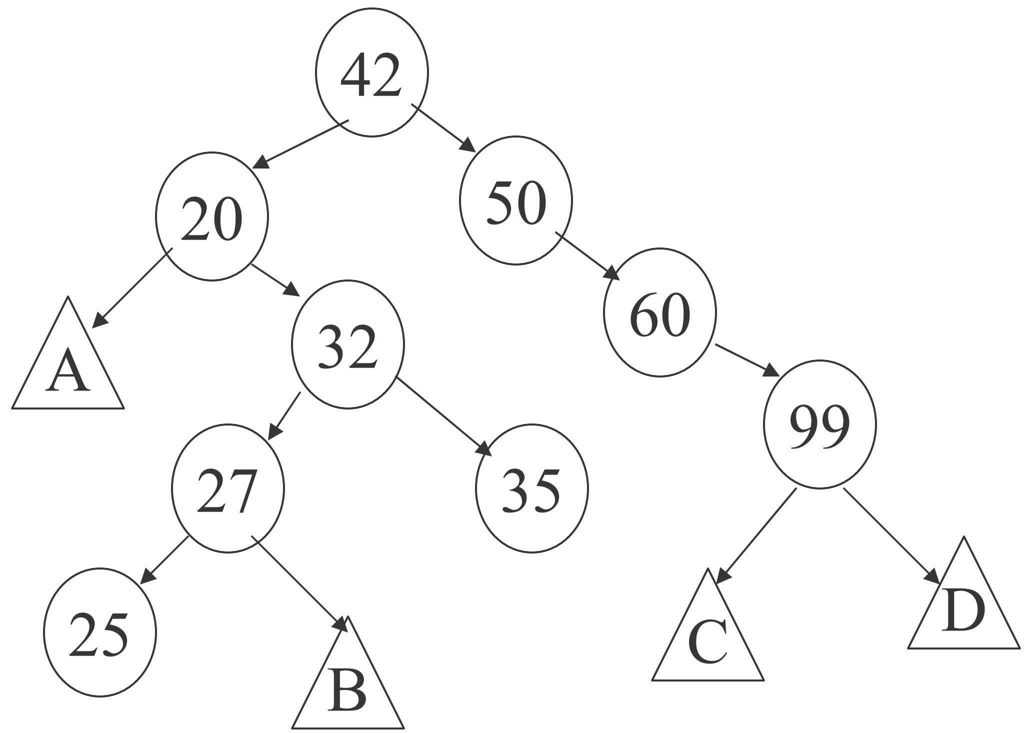
A *Binary Search Tree* is a Binary Tree in which, at every node v, the values stored in the left subtree of v are less than the value at v and the values stored in the right subtree are greater.

The elements in the BST must be comparable.

Duplicates are not allowed in our discussion.

Note that each subtree of a BST is also a BST.

A BST of integers



Describe the values which might appear in the subtrees labeled A, B, C, and D

BST Implementation

The SearchTree ADT

- A *search tree* is a binary search tree which stores homogeneous elements with no duplicates.
- It is dynamic.
- The elements are ordered in the following ways
  - inorder -- as dictated by operator<
  - preorder, postorder, levelorder -- as dictated by the structure of the trer

BST Implementation

```
template <typename Comparable>
class BinarySearchTree
{
  public:
    BinarySearchTree( );
    BinarySearchTree( const BinarySearchTree & rhs );
    ~BinarySearchTree( );

    const Comparable & findMin( ) const;
    const Comparable & findMax( ) const;
    bool contains( const Comparable & x ) const;
    bool isEmpty( ) const;
    void printTree( ) const;

    void makeEmpty( );
    void insert( const Comparable & x );
    void remove( const Comparable & x );
```

BST Implementation (2)

```
    const BinarySearchTree &
          operator=( const BinarySearchTree & rhs );

private:
    struct BinaryNode
    {
      Comparable element;
      BinaryNode *left;
      BinaryNode *right;

      BinaryNode( const Comparable & theElement,
                  BinaryNode *lt, BinaryNode *rt )
       :element( theElement ), left( lt ), right( rt)
          { }
    };
```

BST Implementation (3)

```
   // private data
    BinaryNode *root;

   // private recursive functions
   void insert( const Comparable & x, BinaryNode * & t ) const;
   void remove(const Comparable & x, BinaryNode * & t ) const;
   BinaryNode * findMin( BinaryNode *t ) const;
   BinaryNode * findMax( BinaryNode *t ) const;
   bool contains( const Comparable & x, BinaryNode *t ) const;
   void makeEmpty( BinaryNode * & t );
   void printTree( BinaryNode *t ) const;
   BinaryNode * clone( BinaryNode *t ) const;
};
```

BST "contains" method

```
// Returns true if x is found (contained) in the tree.
bool contains( const Comparable & x ) const
{
  return contains( x, root );
}

// Internal (private) method to test if an item is in a subtree.
//      x is item to search for.
//      t is the node that roots the subtree.
bool contains( const Comparable & x, BinaryNode *t ) const
{
   if( t == NULL )
        return false;
   else if( x < t->element )
        return contains( x, t->left );
   else if( t->element < x )
        return contains( x, t->right );
   else
        return true;    // Match
}
```

Performance of “contains”

Searching in randomly built BST is O(lg n) on average

- but generally, a BST is not randomly built

Asymptotic performance is O(height) in all cases

The insert Operation

```
 // Internal method to insert into a subtree.
 //     x is the item to insert.
 //     t is the node that roots the subtree.
 //     Set the new root of the subtree.

void insert( const Comparable & x, BinaryNode * & t )
{
   if( t == NULL )
        t = new BinaryNode( x, NULL, NULL );
   else if( x < t->element )
        insert( x, t->left );
   else if( t->element < x )
        insert( x, t->right );
   else
        ;  // Duplicate; do nothing
}
```

Predecessor in BST

Predecessor of a node v in a BST is the node that holds the data value that immediately precedes the data at v in order.

Finding predecessor

- v has a left subtree
  - then predecessor must be the largest value in the left subtree (the rightmost node in the left subtree)
- v does not have a left subtree
  - predecessor is the first node on path back to root that does not have v in its left subtree

Successor in BST

Successor of a node v in a BST is the node that holds the data value that immediately follows the data at v in order.

Finding Successor

- v has right subtree
  - successor is smallest value in right subtree (the leftmost node in the right subtree)
- v does not have right subtree
  - successor is first node on path back to root that does not have v in its right subtree

The remove Operation

```
 // Internal (private) method to remove from a subtree.
 //     x is the item to remove.
 //            t is the node that roots the subtree.
 // Set the new root of the subtree.
void remove( const Comparable & x, BinaryNode * & t )
{
   if( t == NULL )
        return;                // x not found; do nothing
   if( x < t->element )
        remove( x, t->left );
   else if( t->element < x )
        remove( x, t->right );
   else if( t->left != NULL && t->right != NULL ) // two children
        {
            t->element = findMin( t->right )->element;
            remove( t->element, t->right );
        }
        else  // zero or one child
        {
            BinaryNode *oldNode = t;
            t = ( t->left != NULL ) ? t->left : t->right;
            delete oldNode;
        }
  }
```

Implementation of makeEmpty

```
template <typename Comparable>
void BinarySearchTree<Comparable>::
makeEmpty( )                       // public makeEmpty ( )
{
   makeEmpty( root );              // calls private makeEmpty ( )
}

template <typename Comparable>
void BinarySearchTree<Comparable>::
makeEmpty( BinaryNode<Comparable> *& t ) const
 {
   if ( t != NULL ) {              // post order traversal
        makeEmpty ( t->left );
        makeEmpty ( t->right );
        delete t;
   }
   t = NULL;
}
```

Implementation of Assignment Operator

```
 // operator= makes a deep copy via cloning
const BinarySearchTree & operator=( const BinarySearchTree & rhs )
{
   if( this != &rhs )
   {
        makeEmpty( );                    // free LHS nodes first
         root = clone( rhs.root );       // make a copy of rhs
    }
    return *this;
}

//Internal method to clone subtree -- note the recursion
BinaryNode * clone( BinaryNode *t ) const
{
   if( t == NULL )
        return NULL;
   return new BinaryNode(t->element, clone(t->left), clone(t->right);
}
```

Performance of BST methods

What is the asymptotic performance of each of the BST methods?

	Best Case	**Worst Case**	**Average Case**
contains			
insert			
remove			
findMin/Max			
makeEmpty			
assignment			

Building a BST

Given an array/vector of elements, what is the performance (best/worst/average) of building a BST from scratch?

Tree Iterators

As we know there are several ways to traverse through a BST. For the user to do so, we must supply different kind of iterators. The iterator type defines how the elements are traversed.

- `InOrderIterator<T> *InOrderBegin( );`
- `PerOrderIterator<T> *PreOrderBegin( );`
- `PostOrderIterator<T> *PostOrderBegin ( );`
- `LevelOrderIterator<T> *LevelOrderBegin( );`

Using Tree Iterator

```
main ( )
{
   BST<int> tree;

   // store some ints into the tree

   BST<int>::InOrderIterator<int> itr = tree.InOrderBegin( );
   while ( itr != tree.InOrderEnd( ) )
   {
        int x = *itr;

        // do something with x

        ++itr;

   }
}
```

BST begin( ) and end( )

```
// BST InOrderBegin( ) to create an InOrderIterator
template <typename T>
InOrderIterator<T> BST<T>::InOrderBegin( )
{
  return InOrderIterator( m_root );
}

// BST InOrderEnd( ) to signal "end" of the tree
template <typename T>
InOrderIterator<T> BST<T>::InOrderBegin( )
{
  return InOrderIterator( NULL );
}
```

Iterator Class with a List
The InOrderIterator is a disguised List Iterator

```
// An InOrderIterator that uses a list to store
// the complete in-order traversal
template < typename T >
class InOrderIterator
{
   public:
        InOrderIterator( );
        InOrderIterator operator++ ( );
        T operator* ( ) const;
        bool operator != (const InOrderIterator& rhs) const;
   private:
        InOrderIterator( BinaryNode<T> * root);
        typename List<T>::iterator m_listIter;
        List<T> m_theList;
};
```

```
// InOrderIterator constructor
// if root == NULL, an empty list is created
template <typename T>
InOrderIterator<T>::InOrderIterator( BinaryNode<T> * root )
{
    FillListInorder( m_theList, root );
    m_listIter = m_theList.begin( );
}

// constructor helper function
template <typename T>
void FillListInorder(List<T>& list, BinaryNode<T> *node)
{
   if (node == NULL) return;
   FillListInorder( list, node->left );
   list.push_back( node->data );
   FillListInorder( list, node->right );
}
```

List-based InOrderIterator Operators
Call List Iterator operators

```
template <typename T>
T InOrderIterator<T>::operator++ ( )
{
   ++m_listIter;
}

template <typename T>
T InOrderIterator<T>::operator* ( ) const
{
   return *m_listIter;
}
template <typename T>
bool InOrderIterator<T>::
operator!= (const InorderIterator& rhs ) const
{
   return m_ListIter != rhs.m_listIter;
}
```

InOrderIterator Class with a Stack

```
// An InOrderIterator that uses a stack to mimic recursive traversal
// InOrderEnd( ) creates a stack containing only a NULL point
// InOrderBegin( ) pushes a NULL onto the stack so that iterators
//  can be compared

template < typename T >
class InOrderIterator
{
   public:
      InOrderIterator( );
      InOrderIterator operator++ ( );
      T operator* ( ) const;
      bool operator== (const InOrderIterator& rhs) const;
  private:
      InOrderIterator( BinaryNode<T>* root );
      Stack<BinaryNode<T> *> m_theStack;
};
```

Stack-Based InOrderIterator Constructor

```
template< typename T >  // default constructor
InOrderIterator<T>::InOrderIterator ( )
{
    m_theStack.Push(NULL);
}

// if t is null, an empty stack is created
template <typename T>
InOrderIterator<T>::InOrderIterator( BinaryNode<T> *t )
{
   // push a NULL as "end" of traversal
   m_theStack.Push( NULL );

   BinaryNode *v = t;          // root
   while (v != NULL) {
      m_theStack.Push(v);   // push root
      v = v->left;          // and all left descendants
   }

}
```

Stack-Based InOrderIterator Operators

```
template <typename T>
InOrderIterator<T> InOrderIterator<T>::operator++( )
{
   if (m_theStack.IsEmpty( ) || m_theStack.Top() == NULL)
          throw StackException( );

   BinaryNode *v = (m_theStack.Top( ))->right;
   m_theStack.Pop();
   while ( v != NULL )
   {
      m_theStack.Push( v );// push right child
      v = v->left;         // and all left descendants
   }
   return *this;
}
```

```
// operator* -- return data from node on top of stack
template< typename T >
T InOrderIterator<T>::operator*( ) const
{
  if (m_theStack.IsEmpty( ))
     throw StackException();
  return (m_theStack.Top())->element;
}

// operator ==
template< typename T>
bool InOrderIterator<T>::
operator== (const InOrderIterator& rhs) const
{
  return m_theStack.Top( ) == rhs.m_theStack.Top( );
}
```

More Recursive Binary (Search) Tree Functions

- `bool isBST ( BinaryNode<T> *t )`
  returns true if the Binary tree is a BST

- `const T& findMin( BinaryNode<T> *t )`
  returns the minimum value in a BST

- `int CountFullNodes ( BinaryNode<T> *t )`
  returns the number of full nodes (those with 2 children) in a binary tree

- `int CountLeaves( BinaryNode<T> *t )`
  counts the number of leaves in a Binary Tree