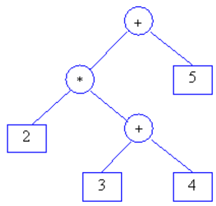
8 Parsing

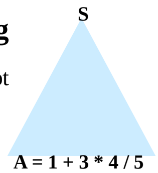


2 * (3 + 4) + 5

Parsing

- A grammar describes syntactically legal strings in a language
- A *recogniser* simply accepts or rejects strings
- A *generator* produces strings
- A *parser* constructs a parse tree for a string
- Two common types of parsers:
  - bottom-up or data driven
  - top-down or hypothesis driven
- A *recursive descent parser* easily implements a top-down parser for simple grammars

Top down vs. bottom up parsing



- The parsing problem is to connect the root node S with the tree leaves, the input
- **Top-down parsers:** starts constructing the parse tree at the top (root) and move down towards the leaves. Easy to implement by hand, but requires restricted grammars. E.g.:
  - Predictive parsers (e.g., LL(k))
- **Bottom-up parsers:** build nodes on the bottom of the parse tree first. Suitable for automatic parser generation, handles larger class of grammars. E.g.:
  - shift-reduce parser (or LR(*k*) parsers)

Top down vs. bottom up parsing

- Both are general techniques that can be made to work for all languages (but not all grammars!)
- Recall that a given language can be described by several grammars
- Both of these grammars describe the same language

```
E -> E + Num
E -> Num
```

```
E -> Num + E
E -> Num
```

- The first one, with it's left recursion, causes problems for top down parsers
- For a given parsing technique, we may have to transform the grammar to work with it

Parsing complexity

- How hard is the parsing task? How to we measure that?
- Parsing an arbitrary CFG is $O(n^3)$ -- it can take time proportional the cube of the number of input symbols
  - This is bad! (why?)
- If we constrain the grammar somewhat, we can always parse in linear time. This is good! (why?)
- Linear-time parsing
  - LL parsers
    - Recognize LL grammar
    - Use a top-down strategy
  - LR parsers
    - Recognize LR grammar
    - Use a bottom-up strategy

- **LL(n) : Left to right, Leftmost derivation, look ahead at most n symbols.**
- **LR(n) : Left to right, Right derivation, look ahead at most n symbols.**

Top Down Parsing Methods

- Simplest method is a full-backup, *recursive descent* parser
- Often used for parsing simple languages
- Write recursive recognizers (subroutines) for each grammar rule
  - If rules succeeds perform some action (i.e., build a tree node, emit code, etc.)
  - If rule fails, return failure. Caller may try another choice or fail
  - On failure it "backs up"

Top Down Parsing Methods: Problems

- When going forward, the parser consumes tokens from the input, so what happens if we have to back up?
  - suggestions?
- Algorithms that use backup tend to be, in general, inefficient
  - There might be a large number of possibilities to try before finding the right one or giving up
- Grammar rules which are left-recursive lead to non-termination!

Recursive Decent Parsing: Example

For the grammar:

```
<term> -> <factor> {(*|/)<factor>}*
```

We could use the following recursive descent parsing subprogram (this one is written in C)

```
void term() {
  factor();      /* parse first factor*/
  while (next_token == ast_code ||
         next_token == slash_code) {
    lexical();   /* get next token */
    factor();    /* parse next factor */
  }
}
```

Problems

- Some grammars cause problems for top down parsers
- Top down parsers do not work with left-recursive grammars
  - E.g., one with a rule like: E -> E + T
  - We can transform a left-recursive grammar into one which is not
- A top down grammar can limit backtracking if it only has one rule per non-terminal
  - The technique of rule factoring can be used to eliminate multiple rules for a non-terminal

Left-recursive grammars

- A grammar is left recursive if it has rules like

  `X -> X β`
- Or if it has indirect left recursion, as in

  `X -> A β`

  `A -> X`
- Q: Why is this a problem?
  - A: it can lead to non-terminating recursion!

Direct Left-Recursive Grammars

- Consider

  `E -> E + Num`

  `E -> Num`
- We can manually or automatically rewrite a grammar removing left-recursion, making it ok for a top-down parser.

Elimination of Direct Left-Recursion

- Consider left-recursive grammar

  `S → S α`

  `S -> β`
- S generates strings

  β

  β α

  β α α ...
- Rewrite using right-recursion

  `S → β S'`

  `S' → α S' | ε`
- Concretely

  `T -> T + id`

  `T-> id`
- T generates strings

  id

  id+id

  id+id+id ...
- Rewrite using right-recursion

  `T -> id`

  `T -> id T`

General Left Recursion

- The grammar

  $S \rightarrow A\ \alpha \mid \delta$

  $A \rightarrow S\ \beta$

  is also left-recursive because

  $S \rightarrow^{+} S\ \beta\ \alpha$

  where $\rightarrow^{+}$ means "can be rewritten in one or more steps"
- This indirect left-recursion can also be automatically eliminated *(not covered)*

Summary of Recursive Descent

- Simple and general parsing strategy
  - Left-recursion must be eliminated first
  - … but that can be done automatically
- Unpopular because of backtracking
  - Thought to be too inefficient
- In practice, backtracking is eliminated by further restricting the grammar to allow us to successfully *predict* which rule to use

Predictive Parsers

- That there can be many rules for a non-terminal makes parsing hard
- A *predictive parser* processes the input stream typically from left to right
  - Is there any other way to do it? Yes for programming languages!
- It uses information from peeking ahead at the *upcoming terminal symbols* to decide which grammar rule to use next
- And *always* makes the right choice of which rule to use
- How much it can peek ahead is an issue

Predictive Parsers

- An important class of predictive parser only peek ahead one token into the stream
- An *LL(k)* parser, does a **L**eft-to-right parse, a **L**eftmost-derivation, and **k**-symbol lookahead
- Grammars where one can decide which rule to use by examining only the *next* token are **LL(1)**
- LL(1) grammars are widely used in practice
  - The syntax of a PL can usually be adjusted to enable it to be described with an LL(1) grammar

Predictive Parser

Example: consider the grammar

S → **if** *E* **then** *S* **else** *S*
S → **begin** *S L*
S → **print** *E*
L → **end**
L → **;** *S L*
E → num = num

An *S* expression starts either with an IF, BEGIN, or PRINT token, and an *L* expression start with an END or a SEMICOLON token, and an *E* expression has only one production.

Remember…

- Given a grammar and a string in the language defined by the grammar …
- There may be more than one way to *derive* the string leading to the *same parse tree*
  - It depends on the order in which you apply the rules
  - And what parts of the string you choose to rewrite next
- All of the derivations are *valid*
- To simplify the problem and the algorithms, we often focus on one of two simple derivation strategies
  - A *leftmost* derivation
  - A *rightmost* derivation

LL(k) and LR(k) parsers

- Two important parser classes are LL(k) and LR(k)
- The name LL(k) means:
  - L: *Left-to-right* scanning of the input
  - L: Constructing *leftmost derivation*
  - k: max # of input symbols needed to predict parser action
- The name LR(k) means:
  - L: *Left-to-right* scanning of the input
  - R: Constructing *rightmost derivation* in reverse
  - k: max # of input symbols needed to select parser action
- A LR(1) or LL(1) parser never need to *"look ahead"* more than *one* input token to know what parser production rule applies

Predictive Parsing and Left Factoring

- Consider the grammar

```
E → T + E
E → T
T → int
T → int * T
T → ( E )
```

Even if left recursion is removed, a grammar may not be parsable with a LL(1) parser

- Hard to predict because
  - For T, two productions start with *int*
  - For E, it is not clear how to predict which rule to use
- Must **left-factor** grammar before use for predictive parsing
- Left-factoring involves rewriting rules so that, if a non-terminal has > 1 rule, each begins with a **terminal**

Left-Factoring Example

Add new non-terminals X and Y to factor out **common prefixes** of rules

```
E → T + E
E → T
T → int
T → int * T
T → ( E )
```

```
E → T X
X → + E
X → ε
T → ( E )
T → int Y
Y → * T
Y → ε
```

For each non-terminal the revised grammar, there is either only one rule or every rule begins with a terminal or ε

Using Parsing Tables

- LL(1) means that for each non-terminal and token there is only **one** production
- Can be represented as a simple table
  - One dimension for current non-terminal to expand
  - One dimension for next token
  - A table entry contains one rule's action or empty if error
- Method similar to recursive descent, except
  - For each non-terminal S
  - We look at the next token *a*
  - And chose the production shown at table cell [S, a]
- Use a stack to keep track of pending non-terminals
- Reject when we encounter an error state, accept when we encounter end-of-input

LL(1) Parsing Table Example

Left-factored grammar

```
E → T X
X → + E | ε
T → ( E ) | int Y
Y → * T | ε
```

End of input symbol

The LL(1) parsing table

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

LL(1) Parsing Table Example

```
E → T X
X → + E | ε
T → ( E ) | int Y
Y → * T | ε
```

- Consider the **[E, int]** entry
  - "When current non-terminal is E & next input *int*, use production E→ T X"
  - It's the only production that can generate an *int* in next place
- Consider the **[Y, +]** entry
  - "When current non-terminal is Y and current token is +, get rid of Y"
  - Y can be followed by + only in a derivation where Y→ε
- Consider the **[E, *]** entry
  - Blank entries indicate error situations
  - "There is no way to derive a string starting with * from non-terminal E"

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

LL(1) Parsing Algorithm

```
initialize stack = <S $> and next
repeat
  case stack of
    <X, rest>  : if T[X,*next] = Y1…Yn
                     then stack ← <Y1… Yn rest>;
                     else  error ();
    <t, rest>  : if t == *next ++
                     then  stack ← <rest>;
                     else error ();
until stack == < >
```

where:
(1) next points to the next input token
(2) X matches some non-terminal
(3) t matches some terminal

LL(1) Parsing Example

Stack	Input	Action
`E $`	`int * int $`	`pop();push(T X)`
`T X $`	`int * int $`	`pop();push(int Y)`
`int Y X $`	`int * int $`	`pop();next++`
`Y X $`	`* int $`	`pop();push(* T)`
`* T X $`	`* int $`	`pop();next++`
`T X $`	`int $`	`pop();push(int Y)`
`int Y X $`	`int $`	`pop();next++;`
`Y X $`	`$`	`pop()`
`X $`	`$`	`pop()`
`$`	`$`	**ACCEPT!**

```
E → TX
X → +E
X → ε
T → (E)
T → int Y
Y → *T
Y → ε
```

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

Constructing Parsing Tables

- No table entry can be multiply defined
- If $A \to \alpha$, where in the line of A do we place α ?
- In column t where t can start a string derived from α
  - $\alpha \to^* t\,\beta$
  - We say that $t \in \text{First}(\alpha)$
- In the column t if α is ε and t can follow an A
  - $S \to^* \beta\, A\, t\, \delta$
  - We say $t \in \text{Follow}(A)$

Computing First Sets

Definition: $\text{First}(X) = \{t \mid X \to^* t\alpha\} \cup \{\varepsilon \mid X \to^* \varepsilon\}$

Algorithm sketch (see book for details):

1. for all terminals t do $\text{First}(t) \leftarrow \{ t \}$
2. for each production $X \to \varepsilon$ do $\text{First}(X) \leftarrow \{ \varepsilon \}$
3. if $X \to A_1 \ldots A_n\ \alpha$ and $\varepsilon \in \text{First}(A_i)$, $1 \le i \le n$ do add $\text{First}(\alpha)$ to $\text{First}(X)$
4. for each $X \to A_1 \ldots A_n$ s.t. $\varepsilon \in \text{First}(A_i)$, $1 \le i \le n$ do add ε to $\text{First}(X)$
5. repeat steps 4 and 5 until no First set can be grown

First Sets. Example

Recall the grammar

$E \to T\,X$ $\qquad X \to +\,E \mid \varepsilon$

$T \to (\,E\,) \mid \text{int}\ Y$ $\qquad Y \to *\,T \mid \varepsilon$

First sets

First(( ) = { (}	First(T) = {int, (}
First( )) = {) }	First(E) = {int, (}
First(int) = { int }	First(X) = {+, ε }
First(+) = { + }	First(Y) = {*, ε }
First(*) = { * }	

Computing Follow Sets

- Definition:

  $\text{Follow}(X) = \{ t \mid S \to^* \beta\, X\, t\, \delta \}$

- Intuition
  - If S is the start symbol then $\$ \in \text{Follow}(S)$
  - If $X \to A\ B$ then $\text{First}(B) \subseteq \text{Follow}(A)$ and $\text{Follow}(X) \subseteq \text{Follow}(B)$
  - Also if $B \to^* \varepsilon$ then $\text{Follow}(X) \subseteq \text{Follow}(A)$

Computing Follow Sets

Algorithm sketch:

1. Follow(S) ← { $ }
2. For each production $A \to \alpha X \beta$
   - add First(β) - {ε} to Follow(X)
3. For each $A \to \alpha X \beta$ where $\varepsilon \in$ First(β)
   - add Follow(A) to Follow(X)

- repeat step(s) ___ until no Follow set grows

Follow Sets. Example

- Recall the grammar

  $E \to T X$ $X \to + E \mid \varepsilon$

  $T \to (E) \mid \text{int } Y$ $Y \to * T \mid \varepsilon$

- Follow sets

  Follow(+) = { int, (} Follow(*) = { int, (}

  Follow(( ) = { int, (} Follow(E) = {), $}

  Follow(X) = {$,) } Follow(T) = {+,) , $}

  Follow( )) = {+,) , $} Follow(Y) = {+,) , $}

  Follow(int) = {*, +,) , $}

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G
- For each production $A \to \alpha$ in G do:
  - For each terminal t ∈ First(α) do
    - T[A, t] = α
  - If $\varepsilon \in$ First(α), for each t ∈ Follow(A) do
    - T[A, t] = α
  - If $\varepsilon \in$ First(α) and $ ∈ Follow(A) do
    - T[A, $] = α

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1)
- Reasons why a grammar is not LL(1) include
  - G is ambiguous
  - G is left recursive
  - G is not left-factored
- Most programming language grammars are not strictly LL(1)
- There are tools that build LL(1) tables

Bottom-up Parsing

- YACC uses bottom up parsing. There are two important operations that bottom-up parsers use: **shift** and **reduce**
  - In abstract terms, we do a simulation of a Push Down Automata as a finite state automata
- Input: given string to be parsed and the set of productions.
- Goal: Trace a rightmost derivation in reverse by starting with the input string and working backwards to the start symbol