## A review of First-Order Logic

## Using KIF

## Knowledge Interchange Format

Material adapted from Professor Richard Fikes Stanford University

## KIF Syntax and Semantics

- Extended version of first order predicate logic


## KR Language Components

- A logical formalism
- Syntax for wffs
- Vocabulary of logical symbols
- Interpretation semantics for the logical symbols
- E.g., (=> (Person ?x) (= (Gender (Mother ?x)) Female)))
- An ontology
- Vocabulary of non-logical symbols
, Relations, functions, constants
- Axioms restricting the interpretations of the symbols
- E.g., (=> (Person ?x) (= (Gender (Mother ?x)) Female)))
- A proof theory
- Specification of the reasoning steps that are logically sound
- E.g., (=> S1 S2) and S1 entails S2


## Conceptualization

- Universe of discourse
- Set of objects about which knowledge is being expressed
- Object
- Concrete Clyde, my car
- Abstract Justice, 2
- Primitive Resister
- Composite Electric circuit
- Fictional Sherlock Holmes


## Relations and Functions

- Relation
- Set of finite lists of objects
, E.g., Parent: \{(Richard Earl) (Richard Polly) (Debbie Don) ... \}
> Mapping: <list of objects> $\rightarrow$ <truth value>
- Function
- Relation that associates a unique nth element with a given $\mathrm{n}-1$ elements
, E.g, +: $\{(134)(172340)(27101231) \ldots\}$
- Referred to as (arg1, arg2, ... , argk, value)
- Mapping: <list of objects> $\rightarrow$ <object>

Blocks World

- Objects - a, b, c, d, e, table
$\square$



## Blocks World

- Objects
- a, b, c, d, e, table
- Relations
- Above: $\{(\mathrm{ab})(\mathrm{ac})(\mathrm{b} \mathrm{c})(\mathrm{d} \mathrm{e})\}$
- Clear: $\{(\mathrm{a})$ (d) $\}$
- Table: $\{(\mathrm{c})(\mathrm{e})\}$
-Functions
- On: $\{(\mathrm{a} b)(\mathrm{b} c)(\mathrm{de})\}$



## Predicate Calculus - KIF

- Knowledge Base- Collection of sentences
-Sentence- Expression denoting a statement
- Term- Expression denoting an object
- Objects always in the conceptualization
- Words
- Complex numbers
- All finite lists of objects
- All sets of objects
- ^ (bottom)


## Declarative Semantics

- Interpretation -
- <object constant> => <object>
- <logical constant> => <truth value>
<relation constant> => (<tuple of objects> $\rightarrow$ <truth value>)
- <function constant> => (<tuple of objects> $\rightarrow$ <object>)
- Variable assignment -
- <individual variable> => <object>
- <sequence variable> => <finite sequence of objects>
- Semantic value - <term> => <object>

Defined in terms of an interpretation and variable assignment

- Truth value - <sentence> => \{true, false\}

Defined in terms of an interpretation and variable assignment

- Version of a variable assignment
$V^{\prime}$ is a version of a variable assignment $V$ with respect to variables var1, ...,varn if and only if $\mathrm{V}^{\prime}$ agrees with V on all variables except for var1,...,varn.


## Constants, Individual Variables, Function Terms

-Constant- Word
E.g., Fred, Block-A, Justice

- SIV (<constant>) $=\mathrm{I}($ constant $>$ )
$\bullet$ Individual Variable- Word beginning with "?" E.g, ?x, ?The-Murderer
> SIV(<individual variable>) = V(<individual variable>)
- Function Term
- (<function constant> <term>* [<sequence variable>]) E.g., (plus 23) (Father-Of Richard)
- SIV((fn term1 ... termn)) = I(fn)[SIV(term1) ... SIV(termn)]
- $\operatorname{SIV}(($ fn term1 $\ldots$ termn @var $))=$
- I(fn)[SIV(term1) $\ldots$ SIV(termn) | V(@var)]


## List Terms and Set Terms

- List Term
- (listof <term>* [<seqvar>])
E.g., (listof A B C) (listof A ?second @rest)
$>\operatorname{SIV}(($ listof term1 $\ldots$ termn $))=$ <SIV(term1), ..., SIV(termn)>
- SIV ((listof term1 ... termn @var)) =
> <SIV(term1), ..., SIV(termn) | V(@var)>
-Set Term
- (setof <term>* [<seqvar>])
, E.g., (setof A B C) (setof A ?X @Z)
$-\operatorname{SIV}(($ setof term1 $\ldots$ termn $))=\{\operatorname{SIV}($ term1),$\ldots$, SIV(termn) $\}$
- SIV ((setof term1 ... termn @var)) $=$
- $\{\operatorname{SIV}($ term1), ... ,SIV(termn) $\} \mathrm{U}\{\mathrm{x} \mid(\$ \mathrm{i}) \mathrm{x}=\operatorname{SIV}($ nth(@var i) $)\}$


## Logical Terms

- (if <sentence> <term> [<term>])
, E.g, (if (Above A B) A B)
- $\operatorname{SIV}(($ (if sent term $))=$
, $\operatorname{SIV}($ term $)$ when $\operatorname{TIV}($ sent $)=$ true
, ^ otherwise
- SIV((if sent term1 term2)) $=$
, $\operatorname{SIV}($ term1 $)$ when $\operatorname{TIV}($ sent $)=$ true
, $\operatorname{SIV}($ term2) otherwise
- (cond (<sentence> <term>) ... (<sentence> <term>))
, E.g., (cond ((Above A B) A) ((Above B A) B))
- $\operatorname{SIV}(($ cond $($ sent1 term1) $\ldots($ sentn termn $)))=$
, SIV(term1) when TIV(sent1) $=$ true
, $\operatorname{SIV}($ termn $)$ when TIV (sentn) $=$ true
, ^ otherwise


## Quantified Terms

-Set Forming Term- (setofall <term> <sentence>) E.g, (setofall ?block (Above ?block A))
$\operatorname{SIV}(($ setofall term sent $))=\left\{\right.$ SIV $^{\prime}($ term $) \mid \operatorname{TIV}^{\prime}($ sent $)=$ true $\}$ for all versions $\mathrm{V}^{\prime}$ of V wrt the variables in term

- Designator- (the <term> <sentence>)
E.g., (the ?block (Above ?block A))
- $\operatorname{SIV}(($ the term sent $))=$

SIV'(term) when
$\mathrm{V}^{\prime}$ is a version of V wrt the variables in term, and
TIV' (sent) = true, and
SIV' ${ }^{\prime \prime}$ (term) $=$ SIV' (term)
for all versions $\mathrm{V}^{\prime \prime}$ of V such that $\mathrm{TIV}^{\prime \prime}($ sent $)=$ true
$\wedge$ otherwise

## Logical Constants, Equations, Inequalities

- Logical constant
- $\operatorname{Tiv}$ (constant) $=\mathrm{I}$ (constant)
- $\operatorname{Tiv}($ true $)=$ true
- $\operatorname{Tiv}($ false $)=$ false
- Equations - (= <term> <term>)
, E.g, (= (Father Richard) Earl) (= A B)
- $\operatorname{TIV}((=$ term1 term2 $))=$
, true when SIV(term1) and SIV(term2) are the same object
, false otherwise
- Inequalities - (/= <term> <term>)
, E.g, (/= (Father Richard) (Father Bob)) (/=A B)
- $\operatorname{TIV}((/=\operatorname{term} 1$ term 2$))=\operatorname{TIV}((\operatorname{not}(=\operatorname{term} 1$ term2 2$))$


## Relational Sentences

- (<relation constant> <term>* [<sequence variable>])
, E.g, (Parent Richard Earl) (Clear A) (Set-Partition Set1 @Sets)
- $\operatorname{TIV}(($ rel term1 $\ldots$ termn $))=$
, true when I(rel)[SIV(term1), ..., SIV (termn)] is true , false otherwise
- $\operatorname{TIV}($ (rel term1 $\ldots$ termn @var) $)=$
, true when I(rel)[SIV (term1), ..., SIV (termn) | SIV (@var)] is true false otherwise
- (<function constant> <term>* <term>)
, E.g, (Father Richard Earl) (Plus 257 )
- $\operatorname{TIV}($ (fun arg $1 \ldots$ argn val) $)=$
, true when I(fun)[SIV(arg1), ..., SIV (argn)] $=\operatorname{SIV}($ val $)$
, false otherwise


## Logical Sentences: not, and, or

- Negation - (not <sentence>)
, E.g., $(\operatorname{not}(O n A D)) \quad(\operatorname{not}(O n B B))$
- $\operatorname{TIV}(($ not sent $))=$
, true when $\operatorname{TIV}($ sent $)$ is false
false otherwise
- Conjunction - (and <sentence>*)
, E.g., (and (On A B) (On B C))
- $\operatorname{TIV}(($ and $\operatorname{sent} 1 \ldots$ sentn $))=$
, true when TIV(senti) is true for all $\mathrm{i}=1, \ldots, \mathrm{n}$
, false otherwise
- Disjunction - (or <sentence>*)
, E.g., (or (On A D) (On A B))
- $\operatorname{TIV}(($ or sent1 $\ldots$ sentn $))=$
, true when TIV(senti) is true for some $i=1, \ldots, n$
, false otherwise


## Logical Sentences: => <= <=>

- Implication - (=> <sentence>* <sentence>)
- E.g., (=> (On A B) (On B C))
- $\operatorname{TIV}((=>$ ante $1 \ldots$ anten conse $))=$
, true when:
- TIV(antei) is false for some $\mathrm{i}=1, \ldots, \mathrm{n}$; or
- TIV (conse) is true
, false otherwise
$\operatorname{TIV}((=>$ a1 $\ldots$ an c) $)=\operatorname{TIV}(($ or (not a1) $\ldots($ not an $) \mathrm{c}))$
- Implication - (<= <sentence> <sentence>*)
- Equivalence - (<=> <sentence> <sentence>)
- $\operatorname{TIV}((\operatorname{sent} 1<=>\operatorname{sent} 2))=$
, true when TIV (sent1) = TIV (sent2)
, false otherwise
- $\operatorname{TIV}((<=>~ s 1 ~ s 2))=\operatorname{TIV}((\operatorname{and}(=>~ s 1 ~ s 2)(=>~ s 2 ~ s 1)))$


## Universally Quantified Sentences

- (forall <individual variable> <sentence>)
E.g, (forall ?b (not (On ?b ?b)))
$\operatorname{TIV}(($ forall ? var sent $))=$ true when TIV' (sent) = true
for all versions $V^{\prime}$ of $V$ with respect to variable ?var false otherwise
- (forall (<individual variable>*) <sentence>) E.g., (forall (?b1 ?b2) (=> (On ?b1 ?b2) (Above ?b1 ?b2)))
$\operatorname{TIV}(($ forall $(? \operatorname{var} 1 \ldots$ ? varn$)$ sent $))=$ true when $\mathrm{TIV}^{\prime}($ sent $)=$ true
for all versions $\mathrm{V}^{\prime}$ of V with respect to ?var1 ... ?varn false otherwise


## Existentially Quantified Sentences

- (exists <individual variable> <sentence>)
E.g, (forall ?b1 (or (on ?b1 table) (exists ?b2 (On ?b1 ?b2))))
- $\operatorname{TIV}(($ exists ? var sent $))=$
, true when TIV' $($ sent $)=$ true
for some version $\mathrm{V}^{\prime}$ of V with respect to variable ?var false otherwise
- (exists (<individual variable>*) <sentence>)
, E.g., (exists (?b1 ?b2) (and (On ?b1 ?A) (Above ?A ?b2)))
- TIV ((exists (?var1 ... ?varn) sent)) =
, true when TIV'(sent) = true
for some version $\mathrm{V}^{\prime}$ of V with respect to ?var1 ... ?varn
, false otherwise
- forall not in the scope of an exists may be omitted
- E.g, (or (on ?b1 table) (exists ?b2 (On ?b1 ?b2)))



## Domain Conceptualization

- Objects
- Circuits
- Terminals
- Signals
- Gates
- Gate types
- Signal values
- Relations
- Connected: (<terminal> <terminal>)
-Functions
- Type: <gate> $\rightarrow$ <gate type>
$\rightarrow$ In: (<index> <gate>) $\rightarrow$ <input terminal>
> Out: (<index> <gate>) $\rightarrow$ <output terminal>
- Signal: <terminal> $\rightarrow$ <signal value>


## Electronic Circuit Domain Theory

- Connected terminals have the same signal
(=> (Connected ?t1 ?t2) (= (Signal ?t1) (Signal ?t2)))
- Signal at terminal is either on or off
(or (= (Signal ?t) On) (= (Signal ?t) Off))
(or (Signal ?t On) (Signal ?t Off)))
(not (= On Off))
- Connected is commutative
(<=> (Connected ?t1 ?t2) (Connected ?t2 ?t1))


## OR and AND Gates

- OR gate's output is on when any of its inputs are on
(=> (= (Type ?g) OR)
(<=> (= (Signal (Out 1 ?g)) On)
(exists ?i (= (Signal (In ?i ?g)) On)))
- AND gate's output is off when any of its inputs are off (=> (= (Type ?g) AND)
(<=> (= (Signal (Out 1 ?g)) Off)
(exists ?i (= (Signal (In ?i ?g)) Off)))


## XOR and NOT Gates

- XOR gate's output is on when its inputs are different (=> (= (Type ?g) XOR)
(<=> (= (Signal (Out 1 ?g)) On)
(not (= (Signal (In 1 ?g) (Signal (In 2 ?g))))))
$\bullet$ NOT gate's output is different from its inputs
(=> (= (Type ?g) NOT)
(not (= (Signal (Out 1 ?g)) (Signal (In 1 ?g)))))


## Circuit C1 Representation

- Gates

$$
\begin{array}{ll}
\text { (= (Type X1) XOR) } & \text { (= (Type X2) XOR) } \\
\text { (= (Type A1) AND) } & \text { (= (Type A2) AND) } \\
\text { (= (Type O1) OR) } &
\end{array}
$$

- Connections
(Connected (Out 1 X1) (In 1 X2)) (Connected (In 1 C1) (In 1 X1))
(Connected (Out 1 X1) (In 2 A2)) (Connected (In 1 C1) (In 1 A1))
(Connected (Out 1 A2) (In 1 O1)) (Connected (In 2 C1) (In 2 X1))
(Connected (Out 1 A1) (In 2 O1)) (Connected (In 2 C1) (In 2 A1))
(Connected (Out 1 X2) (Out 1 C1)) (Connected (In 3 C1) (In 2 X2))
(Connected (Out 1 O1) (Out 2 C1)) (Connected (In 3 C1) (In 1 A2))


## Knowledge About Knowledge

- KIF represents knowledge about knowledge by allowing expressions to be treated as objects in the universe of discourse
- KIF expressions are lists and can be referred to using the quote operator
(=> (believes John '(material moon bleucheese))
(=> (believes john ?p) (believes mary ?p))
or using the listof operator
(=> (believes John (listof 'material ?x ?y))
(believes Lisa (listof 'material ?x ?y))
- Vocabulary is available for "evaluating" an expression
(= (denotation (listof 'F ?x ?y)) (F ?x ? y) )
(=> (sentence ?p) (true (listof '=> ?p ?p)))


## Big KIF and Little KIF

- That KIF is highly expressive language is a desirable feature; but there are disadvantages.
- complicates job of building fully conforming systems.
- resulting systems tend to be "heavyweight"
- KIF has "conformance categories" representing dimensions of conformance and specifying alternatives within that dimension.
- A "conformance profile" is a selection of alternatives from each conformance category.
- System builders decide upon and adhere to a conformance profile sensible for their applications.


## Conformance Categories and Profiles

- Conformance Categories
- logical form: \{atomic, conjunctive, positive, logical, rule-based, quantified\}
- recursion: yes/no
- terms: \{constants, variables, complex terms\}
- relational variables: yes/no
- Common Conformance Profiles might be
- Databases (ground atomic assertions \& conjunctive forms)
- Datalog
- Relational logic
- First order logic
- Second order logic



## KIF vs ANSI KIF

- KIF is the object of an ANSI Ad Hoc standardization group (X3T2)
- ANSI KIF is somewhat different from previous specs
- No non nanotonic rules
- Allow for possible (future) higheq ader extensions
- Defines a standard infix format for presenting KIF


## KIF Software

- Several KIF lased reasoners in LISP are available from Stanford (e.g., EPILOG).
- IBM's ABE (Agent Building Environment) \& RAISE reasoning engine use KIF as their external language.
- Stanford's Ontolingua uses KIF as its internal language.
- Translators (partial) exist for a number of other KR languages, including LOOM, Classic, CLIPS, Prolog,...
- Parsers for KIF exist which take KIF strings into C++ or Java objects.

