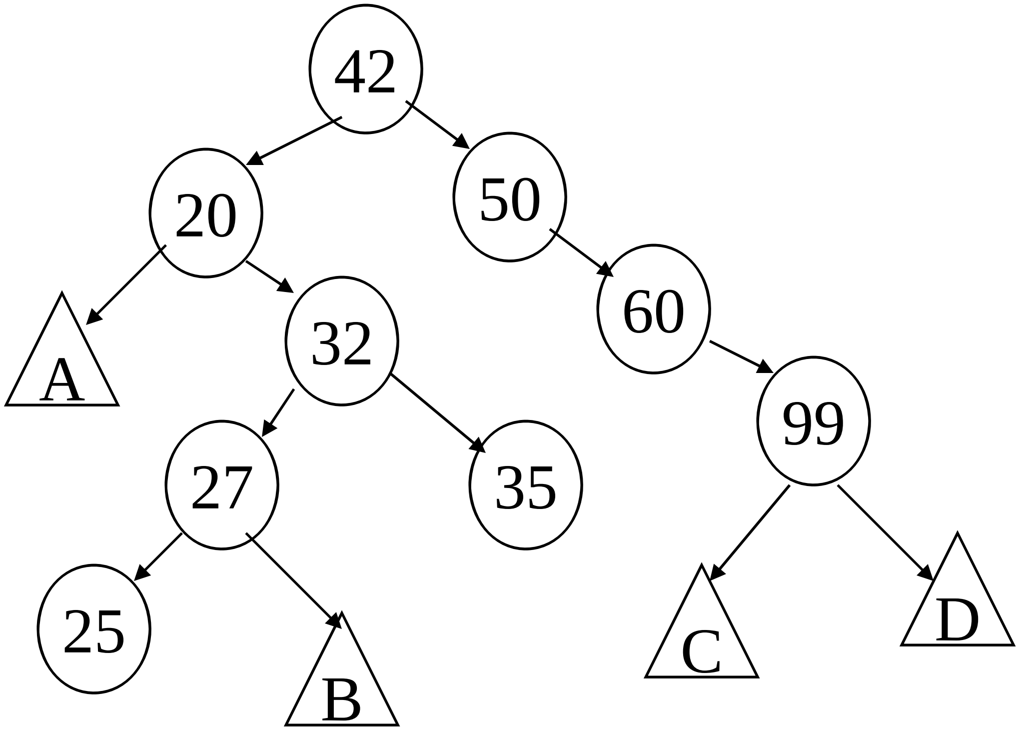
CMSC 341

Binary Search Trees

Binary Search Tree

- **A *Binary Search Tree*** is a Binary Tree in which, at every node v, the values stored in the left subtree of v are less than the value at v and the values stored in the right subtree are greater.
- The elements in the BST must be comparable.
- Duplicates are not allowed in our discussion.
- Note that each subtree of a BST is also a BST.

A BST of integers



Describe the values which might appear in the subtrees labeled A, B, C, and D

SearchTree ADT

- The SearchTree ADT
  - A *search tree* is a binary search tree which stores homogeneous elements with no duplicates.
  - It is dynamic.
  - The elements are ordered in the following ways
    - inorder -- as dictated by operator<
    - preorder, postorder, levelorder -- as dictated by the structure of the tree

BST Implementation

```
public class BinarySearchTree<AnyType extends
                              Comparable<? super AnyType>>
{
    private static class BinaryNode<AnyType>
   {
          // Constructors
          BinaryNode( AnyType theElement )
          { this( theElement, null, null ); }

          BinaryNode( AnyType theElement,
               BinaryNode<AnyType> lt, BinaryNode<AnyType> rt )
          { element  = theElement; left = lt; right = rt; }

          AnyType element;            // The data in the node
          BinaryNode<AnyType> left;   // Left child
          BinaryNode<AnyType> right;  // Right child
    }
```

BST Implementation (2)

```
private BinaryNode<AnyType> root;

public BinarySearchTree( )
{
    root = null;
}

public void makeEmpty( )
{ root = null;
}

public boolean isEmpty( )
{
    return root == null;
}
```

BST “contains” Method

```
 public boolean contains( AnyType x )
{
   return contains( x, root );
}

private boolean contains( AnyType x, BinaryNode<AnyType> t )
{
    if( t == null )
        return false;

    int compareResult = x.compareTo( t.element );

    if( compareResult < 0 )
        return contains( x, t.left );
    else if( compareResult > 0 )
        return contains( x, t.right );
    else
        return true;    // Match
}
```

Performance of “contains”

- Searching in randomly built BST is *O*(lg n) on average
  - but generally, a BST is not randomly built

- Asymptotic performance is *O*(height) in all cases

Implementation of printTree

```
public void printTree()
{
    printTree(root);
}

private void printTree( BinaryNode<AnyType> t )
{
       if( t != null )
       {
           printTree( t.left );
           System.out.println( t.element );
           printTree( t.right );
       }
}
```

BST Implementation (3)

```
public AnyType findMin( )
{
    if( isEmpty( ) ) throw new UnderflowException( );
            return findMin( root ).element;
}
public AnyType findMax( )
{
    if( isEmpty( ) ) throw new UnderflowException( );
            return findMax( root ).element;
}
public void insert( AnyType x )
{
    root = insert( x, root );
}
public void remove( AnyType x )
{
    root = remove( x, root );
}
```

The insert Operation

```
private BinaryNode<AnyType>
insert( AnyType x,  BinaryNode<AnyType> t )
 {
     if( t == null )
         return new BinaryNode<AnyType>( x, null, null );

     int compareResult = x.compareTo( t.element );

     if( compareResult < 0 )
         t.left = insert( x, t.left );
     else if( compareResult > 0 )
         t.right = insert( x, t.right );
     else
         ;  // Duplicate; do nothing
     return t;
 }
```

The remove Operation

```
private BinaryNode<AnyType>
remove( AnyType x,  BinaryNode<AnyType> t )
{
  if( t == null )
       return t;   // Item not found; do nothing
  int compareResult = x.compareTo( t.element );
  if( compareResult < 0 )
       t.left = remove( x, t.left );
  else if( compareResult > 0 )
       t.right = remove( x, t.right );
  else if( t.left != null && t.right != null ){ // 2 children
       t.element = findMin( t.right ).element;
       t.right = remove( t.element, t.right );
  }
  else
       t = ( t.left != null ) ? t.left : t.right;
  return t;
}
```

Implementations of find Max and Min

```
private BinaryNode<AnyType> findMin( BinaryNode<AnyType> t )
{
    if( t == null )
       return null;
    else if( t.left == null )
        return t;
    return findMin( t.left );
}

private BinaryNode<AnyType> findMax( BinaryNode<AnyType> t )
{
    if( t != null )
        while( t.right != null )
            t = t.right;

    return t;
}
```

Average Case Analysis

- Internal Path Length, D(N) =sum of the depths of all nodes in a tree with N nodes
- Consider a tree with the left subtree with i nodes and right subtree with N-i-1 nodes

- D(N)=D(i)+D(N-i-1)+N-1

Replacing D(i) and D(N - i -1) by the average internal path length

$$D(N) = \frac{2}{N}\sum_{j=0}^{N-1} D(j) + N - 1$$

$$D(1) = 0$$

Proof Sketch

Replacing (N - 1) by cN where c is some constant

$$D(N) = \frac{2}{N}\sum_{j=0}^{N-1} D(j) + cN$$

$$ND(N) = 2\sum_{j=0}^{N-1} D(j) + cN^2 \qquad \text{--- Equation 1.}$$

Substituting N - 1 for N in Equation 1,

$$(N-1)D(N-1) = 2\sum_{j=0}^{N-2} D(j) + c(N-1)^2 \qquad \text{--- Equation 2.}$$

Subtracting Equation 2 from Equation 1,

$$ND(N) = (N+1)D(N-1) + 2cN \qquad \text{--- Equation 3}$$

Continued

Dividing both sides of Equation 3 by N(N - 1)

$$\frac{D(N)}{N+1} = \frac{D(N-1)}{N} + \frac{2c}{N+1}$$

$$\frac{D(N-1)}{N} = \frac{D(N-2)}{N-1} + \frac{2c}{N}$$

....

....

$$\frac{D(2)}{3} = \frac{D(1)}{2} + \frac{2c}{3}$$

Continued

Adding all the equations in the previous slide,

$$\frac{D(N)}{N+1} = \frac{D(1)}{2} + 2c\sum_{i=3}^{N+1}\frac{1}{i}$$

$$D(N) = 2c(N+1)\sum_{i=3}^{N+1}\frac{1}{i}$$

$$D(N) = 2c(N+1)O(\lg N)$$

$$D(N) = O(N \lg N)$$

$$\text{Average height} = \frac{\mathrm{O(N\,lg\,N)}}{N} = O(\lg N)$$

Performance of BST methods

- What is the asymptotic performance of each of the BST methods?

	Best Case	**Worst Case**	**Average Case**
contains			
insert			
remove			
findMin/ Max			
makeEmpty			
assignment			

Predecessor in BST

- Predecessor of a node v in a BST is the node that holds the data value that immediately precedes the data at v in order.

- Finding predecessor
  - v has a left subtree
    - then predecessor must be the largest value in the left subtree (the rightmost node in the left subtree)
  - v does not have a left subtree
    - predecessor is the first node on path back to root that does not have v in its left subtree

Successor in BST

- Successor of a node v in a BST is the node that holds the data value that immediately follows the data at v in order.

- Finding Successor
  - v has right subtree
    - successor is smallest value in right subtree (the leftmost node in the right subtree)
  - v does not have right subtree
    - successor is first node on path back to root that does not have v in its right subtree

Building a BST

- Given an array/vector of elements, what is the performance (best/worst/average) of building a BST from scratch?

Tree Iterators

- As we know there are several ways to traverse through a BST. For the user to do so, we must supply different kind of iterators. The iterator type defines how the element are traversed.
  - `InOrderIterator<T>` **`inOrderIterator`**`();`
  - `PreOrderIterator<T>` **`preOrderIterator`**`();`
  - `PostOrderIterator<T>` **`postOrderIterator`**`();`
  - `LevelOrderIterator<T>` **`levelOrderIterator`**`();`

Using Tree Iterator

```
public static void main (String args[] )
{
      BinarySearchTree<Integer> tree = new
                        BinarySearchTree<Integer>();

      // store some ints into the tree

      InOrderIterator<Integer> itr =
                        tree.inOrderIterator( );
      while ( itr.hasNext( ) )
      {
            Object x = itr.next();
            // do something with x
      }
}
```

The InOrderIterator is a Disguised List Iterator

```
// An InOrderIterator that uses a list to store
// the complete in-order traversal
import java.util.*;
class InOrderIterator<T>
{
   Iterator<T> _listIter;
   List<T> _theList;

   T next()
   {    /*TBD*/        }

   boolean hasNext()
   {    /*TBD*/        }

   InOrderIterator(BinarySearchTree.BinaryNode<T> root)
   {    /*TBD*/        }

}
```

List-Based InOrderIterator Methods

```
//constructor
InOrderIterator( BinarySearchTree.BinaryNode<T> root )
{
  fillListInorder( _theList, root );
  _listIter = _theList.iterator( );
}

// constructor helper function
void fillListInorder (List<T> list,
                    BinarySearchTree.BinaryNode<T> node)
{
   if (node == null) return;
   fillListInorder( list, node.left );
   list.add( node.element );
   fillListInorder( list, node.right );
}
```

List-based InOrderIterator Methods Call List Iterator Methods

```
T next()
{
	return _listIter.next();
}

boolean hasNext()
{
	return _listIter.hasNext();
}
```

InOrderIterator Class with a Stack

```
// An InOrderIterator that uses a stack to mimic recursive traversal
class InOrderIterator
{
       Stack<BinarySearchTree.BinaryNode<T>> _theStack;

       //constructor
       InOrderIterator(BinarySearchTree.BinaryNode<T> root){
              _theStack = new Stack();
              fillStack( root );
       }

       // constructor helper function
       void fillStack(BinarySearchTree.BinaryNode<T> node){
               while(node != null){
                     _theStack.push(node);
                     node = node.left;
              }
       }
```

Stack-Based InOrderIterator

```
        T next(){
                BinarySearchTree.BinaryNode<T> topNode = null;
                try {
                        topNode = _theStack.pop();
                }catch (EmptyStackException e)
                {
                        return null;
                }
                if(topNode.right != null){
                        fillStack(topNode.right);
                }
                return topNode.element;
        }

        boolean hasNext(){
                return !_theStack.empty();
        }
}
```

More Recursive BST Methods

- `bool` **`isBST`** `( BinaryNode<T> t )`
  returns true if the Binary tree is a BST
- `const T&` **`findMin`**`( BinaryNode<T> t )`
  returns the minimum value in a BST
- `int` **`countFullNodes`** `( BinaryNode<T> t )`
  returns the number of full nodes (those with 2 children) in a binary tree
- `int` **`countLeaves`**`( BinaryNode<T> t )`
  counts the number of leaves in a Binary Tree